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THE TRANSIENT FIELDS OF DIPOLE ANTENNAS
IN THE PRESENCE OF A DIELECTRIC HALF-SPACE

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FOREWORD

This report OSURF number 2467-2, was prepared by the Electro-Science Laboratory, Department of Electrical Engineering, The Ohio State University at Columbus, Ohio. Research was conducted under Contract F44620-67-C-0095 of the Air Force Cambridge Research Laboratories at Bedford, Massachusetts. Dr. John K. Schindler was the Program Monitor for this research.

ABSTRACT

The transient fields produced by vertical and horizontal electric dipoles with impulsive current excitation in the presence of a half-space are considered. For special source and observer configurations, exact, closed form expressions for the fields produced by impulsively excited dipoles above a nondispersive half-space are given. For the dispersive case, estimates of the transient fields from asymptotic approximations are given. A knowledge of the interactions of dipole antennas and a half-space is useful for studies of the background return from the half-space and for studies of the scattering by small targets on or near the half-space.

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THE TRANSIENT FIELDS OF DIPOLE ANTENNAS IN THE PRESENCE OF A DIELECTRIC HALF-SPACE

I.   INTRODUCTION

When a radar target is located on the ground under a vegetation canopy, its scattering characteristics can be greatly altered from those of the same target in free space. The ecology of the ground and vegetation constitute a single entity but it is more convenient to consider separately the effects of the ground and vegetation cover. A Rayleigh region leaf model of the vegetation canopy has been developed,[1] and calculations of the effective constitutive parameters in the 30 to 300 MHz range obtained. In this frequency range the effects of the ground can often be estimated by treating the ground as a smooth dielectric surface. It is appropriate therefore to consider the fields produced by dipole antennas in the presence of a half-space. Since the dipoles may be used to represent either sources or small scatterers, a knowledge of the interactions of the dipole with the half-space yields estimates of both the background return from the ground and the effects of the ground on the scattering characteristics of a target.

A general transient excitation of the dipoles is considered because: (1) a knowledge of the ground return for a general transient excitation may prove useful in deciding what specific interrogating waveform or frequency spectrum will minimize the problems of background return, and (2) a general extension of the time domain concepts developed for free space targets[2] to the problem of targets located in a natural environment is being studied.

The first two sections of this report consider, respectively, the transient fields of vertical and horizontal dipoles above a nondispersive half-space. For the vertical dipole, exact, closed form solutions for the transient fields when the dipole is impulsively excited are given when the observer is on the vertical axis containing the source dipole or when both source dipole and observer are on the interface. These results are carried out from an anisotropic half-space where the medium has a uniaxial permittivity. For the impulsively excited horizontal dipole, exact, closed form solutions when the observer is on the vertical axis containing the dipole are given. For simplicity, these results are restricted to an isotropic half-space and observation points above the half-space. These simplifications are not necessary, however, and the results can be extended to an anisotropic half-space and observation points in the lower medium.

If the half-space is lossy or stratified, exact solutions for the transient fields are not possible. The third section of this report considers long and short time estimates of the transient fields obtained from quasi-static and asymptotic approximations. Both analytical estimates and a Fourier synthesis technique[2] are used to illustrate some of the effects of a finite conductivity on the transient waveforms.

The final section of the report discusses practical applications of the theoretical results in predicting the effects of the ground on the scattering characteristics of targets located on the ground. A theoretical model for the scattering by targets located on or above the ground, including interactions, is described. A linear difference equation for the transient waveforms is illustrated, and the application of this difference equation in a detection and discrimination scheme is shown. Research on the problem is continuing on Contract F19628-67-C-0239, and additional results will be reported there.

II.   VERTICAL ELECTRIC DIPOLE

A vertical electric dipole located at a height h above an anisotropic half-space is shown in Fig. 1. It is assumed that the half-space $z > 0$ is free space and the half-space $z < 0$ is a uniaxial material with a complex conductivity tensor

$$(1) \qquad (\sigma) = \begin{pmatrix} \sigma_v & 0 & 0 \\ 0 & \sigma_h & 0 \\ 0 & 0 & \sigma_h \end{pmatrix}$$

where

$$\sigma_v = \hat{\sigma}_v + j \omega \epsilon_v$$

$$\sigma_h = \hat{\sigma}_h + j \omega \epsilon_h \qquad .$$

The subscripts h and v refer to the horizontal and vertical directions respectively. The problem corresponds to that considered by Wait[3] when the anisotropic intermediate layer considered by Wait has an infinite depth. It can be shown that[4] for the special case of a nondispersive half-space, i.e., a half-space which can be characterized by frequency independent indices of refraction n_h and n_v, and an observation point located on the z axis (directly above or below the source

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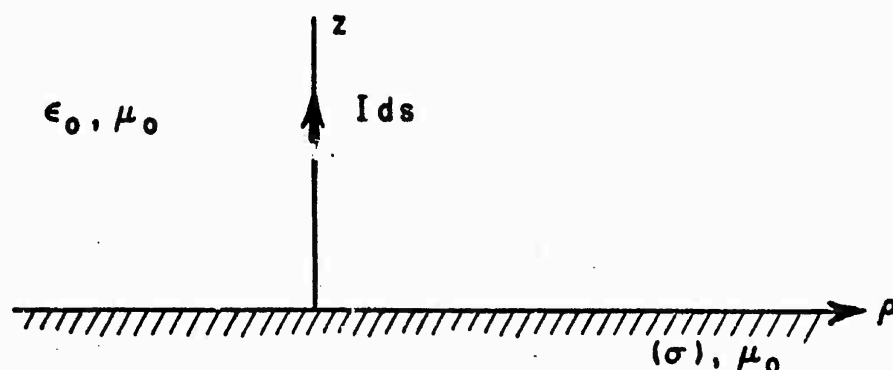
Fig. 1. Dipole over anisotropic half-space.

dipole) that the real time-dependent electric field corresponding to an excitation of the source dipole of length ds with a current $I\delta(t)$ is given by

$$(2) \qquad \tilde{E}^s_{oz}(z, t) = \frac{Ids}{4\pi\epsilon_0(z+h)} \left[\frac{2}{c(z+h)} \frac{n_h-1}{n_h+1} \ \delta\left(t - \frac{z+h}{c}\right) + D(z, t)u\left(t - \frac{z+h}{c}\right)\right],$$

$$z \geq 0$$

where the dipole is of length ds and the quantity $D(z, t)$ is given by

$$(3) \qquad D(z, t) = \frac{2n_h n_v}{(z+h)^2 B^3} \left\{ B\left[\frac{T}{A} - \frac{T^3}{A^3} + 4TA - 5\frac{T^3}{A} + \frac{T^5}{A^3}\right] \right.$$

$$\left. + 2n_v n_h \ A(1 - T^2) - \frac{2T}{A^2}(1 - T^2) - \frac{2T}{A} F(1 - T^2)\right\} + \frac{2}{(z+h)^2} \frac{F}{B} ,$$

where

$$T = \frac{ct}{z+h} \qquad\qquad n_v = \sqrt{\frac{\epsilon_v}{\epsilon_o}} \quad , \ \hat{\sigma}_v = 0$$

$$A = \sqrt{n_v^2 - 1 + T^2} \qquad n_h = \sqrt{\frac{\epsilon_h}{\epsilon_o}} \quad , \ \hat{\sigma}_h = 0$$

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$$B = n_v n_h T + A \quad , \quad \text{and}$$

$$F = n_v n_h T - A \quad .$$

In Eq. (2), the direct term, i.e., the term present in the absence of the half-space, has been removed and the observation point is restricted to the half-space $z \geq 0$. The refractive indices n_h and n_v are defined respectively by the ratio of horizontal or vertical propagation constants to the free space propagation constant. The scattered or interaction field in Eq. (2) is exact for a nondispersive half-space and an observation point on the vertical axis.

The initial and final values of the expression in Eq. (3) are easily obtained as

$$(4) \qquad D\left(z, \frac{z+h}{c}\right) = \frac{2}{(z+h)^2} \left\{ \frac{4n_h(n_v^2 - 1)}{[n_v(n_h+1)]^2} + \frac{n_h-1}{n_h+1} \right\} \quad ,$$

and

$$(5) \qquad D(z, \infty) = \frac{2}{(z+h)^2} \cdot \frac{n_h n_v - 1}{n_h n_v + 1} \quad .$$

$D(z, t)$ in Eq. (3) is a monotonically decreasing function with a time constant reoughly equal to $z+h/c$. Thus while the expression in Eq. (2) is not simple, the resultant waveform is a relatively simple function with an initial singularity and jump discontinuity and then a smooth decay to the final value. A sketch of the waveform of Eq. (2) is shown in Fig. 2.

For the special case of a perfectly conducting lower half-space $(n_v = n_h \to \infty)$

$$(6) \qquad \hat{E}_{oz}(z, t) = \frac{Ids}{2\pi\epsilon_o(z+h)^2} \left[\frac{1}{c} \delta\left[t - \frac{z+h}{c}\right] + \frac{u\left[t - \frac{z+h}{c}\right]}{z+h} \right] \quad ,$$

which agrees with an image theory solution.

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$$\frac{4\pi\epsilon_0}{I\,ds}\,\tilde{E}_{oz}^s$$

$$\frac{2}{C(z+h)^2}\;\frac{n_h-1}{n_h+1}$$

$$\frac{2}{(z+h)^3}\left\{\frac{4n_h(n_v^2-1)}{[n_v(n_h+1)]^2}+\frac{n_h-1}{n_h+1}\right\}$$

$$\frac{2}{(z+h)^3}\;\frac{n_vn_h-1}{n_vn_h+1}$$

$$\frac{ct}{z+h}$$
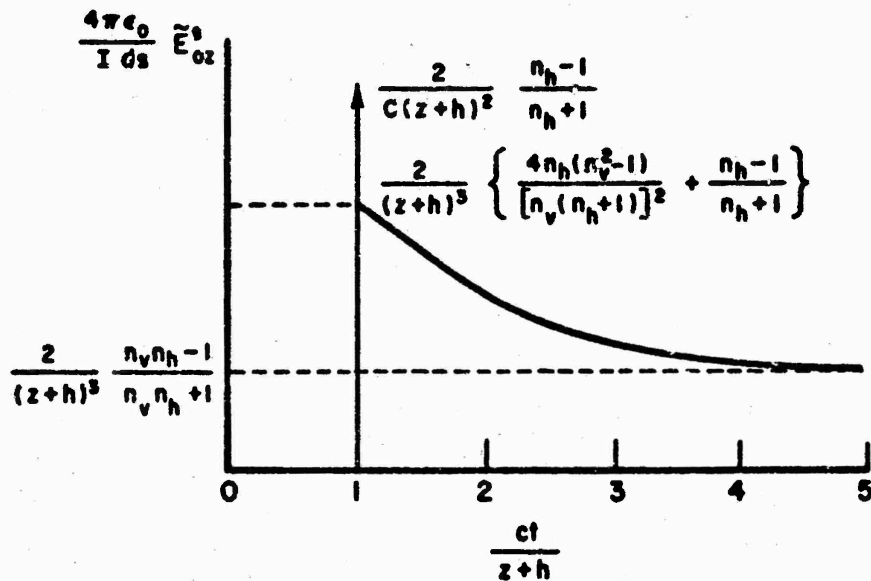
Fig. 2. Sketch of transient field of impulsively excited
vertical electrical dipole above half-space.
Observation point on vertical axis above half-
space.

The background return from the anisotropic half-space is given
by setting $z = h$ in Eq. (2). Note that this result may also be viewed as
the interaction of a vertical electric dipole at a height, h, and an aniso-
tropic half-space.

The general method used to obtain the result in Eq. (2) can also
be applied for observation points on the vertical axis in the lower half-
space $(z < 0)$.[4] In this case an isotropic lower half-space $(n_v = n_h = n)$
is considered although this simplification is not necessary to obtain an
exact, closed form solution for the time-dependent field. It can be
shown that the time-dependent Hertz vector in the lower medium for
observation points on the vertical axis is given by

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$$(7) \qquad \tilde{\Pi}_{1z}(z, t) = \frac{Ids}{2\pi\epsilon_O} \left\{ \frac{z^2 + h^2}{h^2 - z^2} + zh \left(\frac{\dfrac{2}{h^2 - z^2} + \dfrac{n^2 + 1}{(ct)^2}}{\sqrt{1 + (n^2 - 1)\dfrac{(h^2 - z^2)}{c^2 t^2}}} \right) \right\}$$

$$\cdot \left[z + n^2 h + (h + n^2 z)\sqrt{1 + (n^2 - 1)(h^2 - z^2)/(ct)^2} \right]^{-1} u\left(t - \frac{h - nz}{c} \right) .$$

The time-dependent electric field given by

$$(8) \qquad \tilde{E}_{1z}(z, t) = \left[\frac{\partial^2}{\partial z^2} - \left(\frac{n}{c} \right)^2 \frac{\partial^2}{\partial t^2} \right] \tilde{\Pi}_{1z}(z, t) ,$$

is quite complicated, as is obvious from Eq. (7), and will not be given here. An exact, closed form expression for this field, however, can be obtained from the results in Eqs. (7) and (8). The final value of the field is given by

$$(9) \qquad \tilde{E}_{1z}(z, \infty) = \frac{Ids}{\pi\epsilon_O (h - z)^3 (n^2 + 1)} .$$

A sketch of the Hertz vector given in Eq. (7) is shown in Fig. 3. Note that this waveform is monotonic decreasing and relatively simple despite the complexity of Eq. (7). It follows that the time-dependent field given by Eq. (8) will have singularities and a discontinuity at the waveform origin but will also be monotonic decreasing for longer times.

One other special orientation of source dipole and observer yields an exact, closed form expression for the time-dependent field. If both the source dipole and the observation point lie on the interface (z = h = 0), then it can be shown[4] that

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$$(10) \quad \hat{E}_z(\rho, t) = \frac{-Ids}{2\pi\epsilon_0\rho^2}\left\{\frac{1}{c}\left(\frac{n_v^2 n_h^2}{n_v^2 n_h^2 - 1}\right)\left(\frac{n_h^2 + 1}{(n_h^2 - n_v^2 + 1)^{3/2}}\right)\delta\left[t - \frac{\rho}{c}\right]\right.$$

$$- \frac{1}{c}\frac{n_v^2(n_h^2 + 1)}{n_h(n_v^2 n_h^2 - 1)}\delta\left[t - n_v\frac{\rho}{c}\right] + \frac{1}{\rho}\left(\frac{n_v^2 n_h^2}{n_v^2 n_h^2 - 1}\right)$$

$$\left[\frac{1}{\sqrt{n_h^2 - n_v^2 + 1}} - \frac{2n_v^2}{\left[(n_h^2 + 1)\left(\frac{ct}{\rho}\right)^2 - n_v^2\right]^{3/2}}\right.$$

$$\left.\frac{-3 n_v^4}{\left[(n_h^2 + 1)\left(\frac{ct}{\rho}\right)^2 - n_v^2\right]^{5/2}}\right]\left[u\left(t - \frac{\rho}{c}\right) - u\left(t - n_v\frac{\rho}{c}\right)\right]$$

$$\left. + \frac{1}{\rho}\left(\frac{n_v n_h}{n_v^2 n_h^2 - 1}\right)\left(\frac{n_v n_h}{\sqrt{n_h^2 - n_v^2 + 1}} - 1\right)u\left[t - n_v\frac{\rho}{c}\right]\right\},$$

where ρ is the horizontal separation of the source dipole and observation point. A corresponding expression for the ϕ component of the field (ρ, ϕ, z cylindrical coordinates) is

$$(11) \quad \hat{H}_\phi(\rho, t) = \frac{Ids}{2\pi\rho^2}\left\{\left(\frac{n_v^2 n_h^2}{n_v^2 n_h^2 - 1}\right)\left(\frac{n_h^2 + 1}{(n_h^2 - n_v^2 + 1)^{3/2}}\right)\delta\left[t - \frac{\rho}{c}\right]\right.$$

$$- \frac{n_v(n_h^2 + 1)}{n_h(n_v^2 n_h^2 - 1)}\delta\left[t - n_v\frac{\rho}{c}\right] - \frac{3c}{\rho}\left(\frac{n_v^4 n_h^2(n_h^2 + 1)}{n_v^2 n_h^2 - 1}\right)$$

$$\left.\left(\frac{\frac{ct}{\rho}}{\left[(n_h^2 + 1)\left(\frac{ct}{\rho}\right)^2 - n_v^2\right]^{5/2}}\right)\left[u\left[t - \frac{\rho}{c}\right] - u\left[t - n_v\frac{\rho}{c}\right]\right]\right\}.$$

the ρ component of the electric field cannot be obtained from this procedure because the z dependence of the intermediate Hertz vector is unknown. Van der Pol[5] has obtained similar results for the case of an isotropic half-space.

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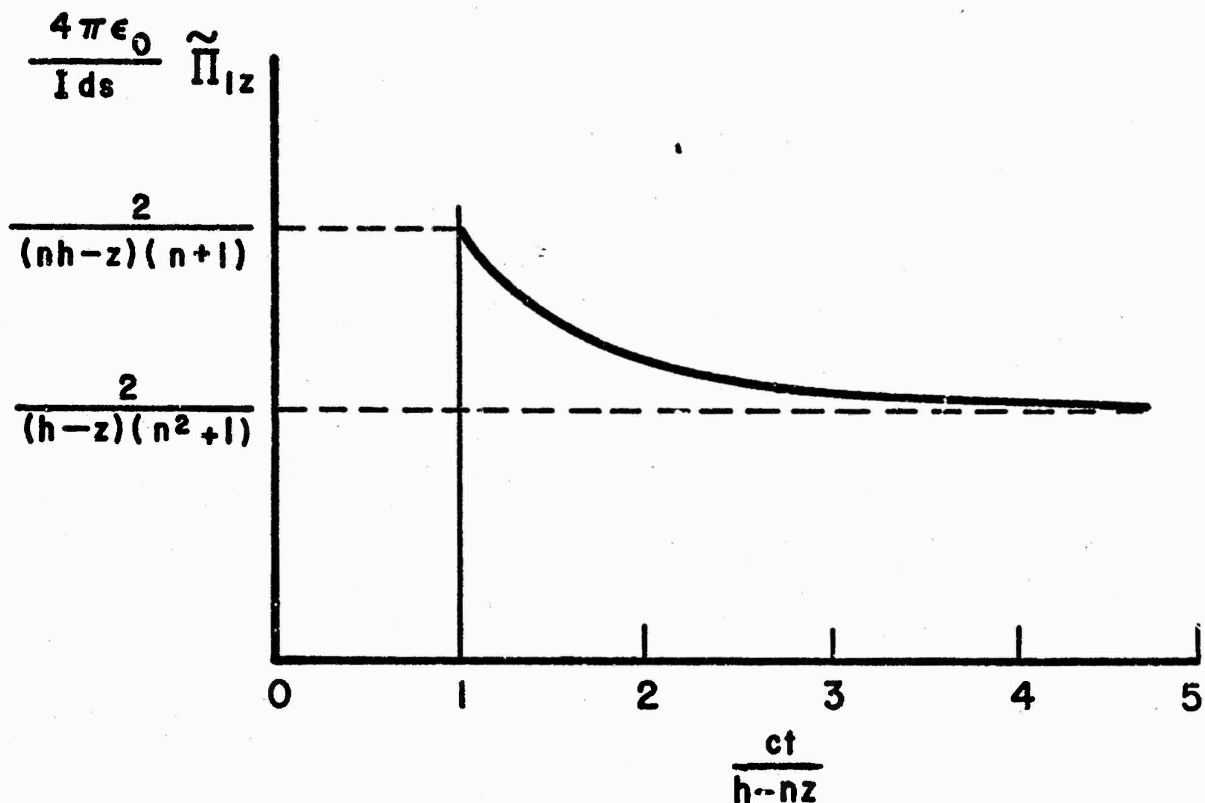
Fig. 3. Sketch of time-dependent Hertz vector of impulsively
excited vertical electric dipole, vertical axis. Source
dipole above half-space, observation point within
half-space.

The results in both Eq. (10) and Eq. (11) show a double wavefront,
i.e., one wave traveling with a velocity c along the boundary and a
second wave traveling with a velocity c/n_v along the boundary. The lead-
ing terms in both Eqs. (10) and (11) have typical ρ^{-2} behaviour for sur-
face waves. Note that the ϕ component of the field (Eq. (11)) has a finite
duration. A sketch of the waveform in Eq. (11) is shown in Fig. 4. The
time-dependent fields given by Eqs. (10) and (11) are typical of the excita-
tion produced by a vertically polarized source at very low grazing angles.
The results in Eqs. (10) and (11) are for the total fields, i.e., the direct
term is included.

III. HORIZONTAL ELECTRIC DIPOLE

If the vertical electric dipole in Fig. 1 is replaced with a hori-
zontal electric dipole, oriented in the x direction, the problem has an
added complexity because of the loss of a ϕ symmetry. It is convenient,

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$$\frac{2\pi}{I\,ds}\tilde{H}'_{o\phi}\qquad \frac{(n_h^2+1)}{(n_h^2-n_v^2+1)^{3/2}\rho^2}\frac{n_v^2 n_h^2}{(n_v^2 n_h^2-1)}$$

$$-\frac{3C}{\rho^3 n_h^3(n_v^2 n_h^2-1)}$$

$$-\frac{3C\,n_v^2 n_h^2}{\rho^3(n_h^2-n_v^2+1)^{5/2}(n_v^2 n_h^2-1)}$$

$$\frac{ct}{\rho}\qquad n_v$$

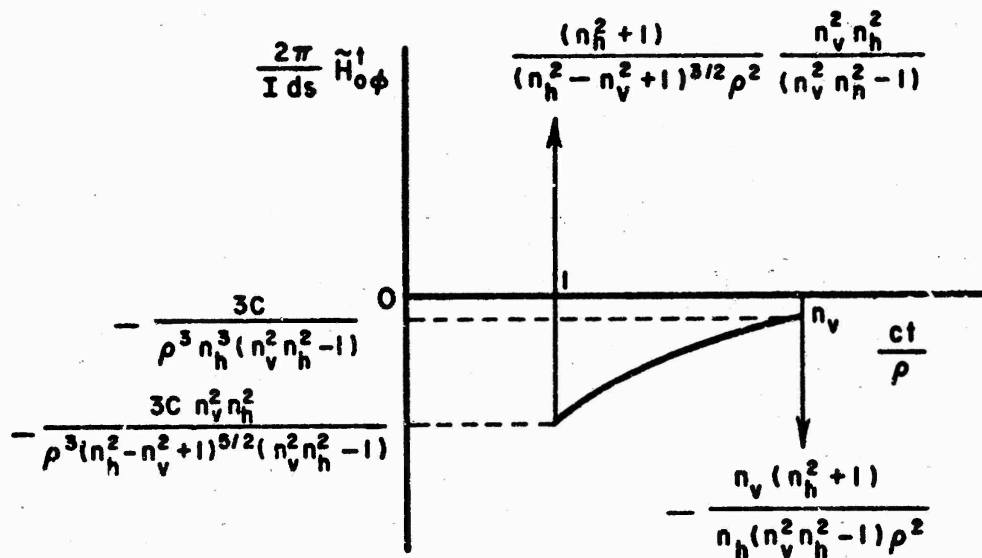$$-\frac{n_v(n_h^2+1)}{n_h(n_v^2 n_h^2-1)\rho^2}$$

Fig. 4. Sketch of transient field of impulsively excited
vertical electric dipole. Source dipole and
observation point on interface.

but unnecessary, to assume an isotropic lower half-space. For a dispersionless half-space and an observation point on the vertical axis in the upper medium ($z \geq 0$), it can be shown that for a dipole of length ds the time-dependent field produced by a current excitation $I\delta(t)$ is given by[4]

$$(12)\qquad \tilde{E}^s_{ox}(z,t) = \frac{Ids}{4\pi\epsilon_0}\left\{\frac{n-1}{n+1}\frac{\delta'\left(t-\frac{z+h}{c}\right)}{c^2(z+h)} + \frac{n-1}{n+1}\frac{\delta\left(t-\frac{z+h}{c}\right)}{c(z+h)^2}\right.$$
$$z \geq 0$$

$$-\frac{\partial^2}{\partial t^2}\left[\frac{2}{1+\sqrt{1+(n^2-1)\left(\frac{z+h}{ct}\right)^2}} + \frac{\left(\frac{ct}{z+h}\right)^2-1}{n^2+\sqrt{1+(n^2-1)\left(\frac{z+h}{ct}\right)^2}}\right]\cdot$$

$$\left.\cdot\frac{u\left[t-\frac{z+h}{c}\right]}{c^2(z+h)} + \frac{1}{(z+h)^3}\,u\left[t-\frac{z+h}{c}\right]\right\}.$$

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$$
\begin{aligned}
(13)\quad \tilde{H}^s_{oy}(z,t) &= \frac{Ids}{4\pi}\left\{\frac{n-1}{n+1}\;\frac{\delta'\!\left[t-\dfrac{z+h}{c}\right]}{c(z+h)} + \frac{n-1}{n+1}\;\frac{\delta\!\left[t-\dfrac{z+h}{c}\right]}{(z+h)^2}\right.\\[2mm]
z\ge 0 \qquad\\[2mm]
&\quad+\frac{\partial}{\partial z}\left\{\frac{\partial}{\partial t}\left[\frac{2\left(\dfrac{ct}{z+h}\right)}{\dfrac{ct}{z+h}+\left(\dfrac{ct}{z+h}\right)^2+n^2-1}\right.\right.
\end{aligned}
$$

$$
\left.\left.+\,(n^2+1)\;\frac{\left(\dfrac{ct}{z+h}\right)^3-\left(\dfrac{ct}{z+h}\right)}{n^2\left(\dfrac{ct}{z+h}\right)+\left(\dfrac{ct}{z+h}\right)^2+n^2+1}\right]-\frac{2c}{z+h}\left(\dfrac{ct}{z+h}\right)\right\}\cdot
$$

$$
\cdot\,\frac{u\!\left[t-\dfrac{z+h}{c}\right]}{z+h}\Bigg\}
$$

The final value of the magnetic field is zero and the final value of the electric field is given by

$$
(14)\qquad \tilde{E}^s_{ox}(z,\infty) = \frac{Ids}{4\pi\epsilon_0(z+h)^3}\;\frac{n^2-1}{n^2+1}\quad.
$$

A sketch of the waveform given in Eq. (13) is shown in Fig. 5. For the results in Eqs. (12) and (13), the direct term has been removed. Similar results under the same restrictions for the time-dependent fields in the half-space $z < 0$ can be obtained, and the results for the horizontal dipole can also be extended to an anisotropic half-space.

Details of the methods used to obtain the results for the vertical and horizontal dipoles are given in Reference 4 and will be included in a future report.[11] Basically, the inverse Fourier transform of the Sommerfeld-type integral solutions is taken after a change of variable. It turns out that for a nondispersive half-space, the inverse transform may be obtained for vertical or horizontal, electric or magnetic dipoles, for observation points in either half-space, assuming an interchange of orders of integration is valid. For arbitrary locations of the source dipole and observer, the method results in a remaining finite integral which must be evaluated numerically. For the special cases considered here, the remaining integral can be evaluated exactly in closed form.

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$$\frac{4\pi\epsilon_0}{I\,ds}\ \tilde{E}_{0x}^0$$

$$\frac{1}{C(z+h)^2}\frac{n-1}{n+1}$$

$$\frac{1}{(z+h)^3}\,f(n)$$

$$\frac{1}{(z+h)^3}\frac{n^2-1}{n^2+1}$$

$$\frac{1}{C^2(z+h)}\frac{n-1}{n+1}$$

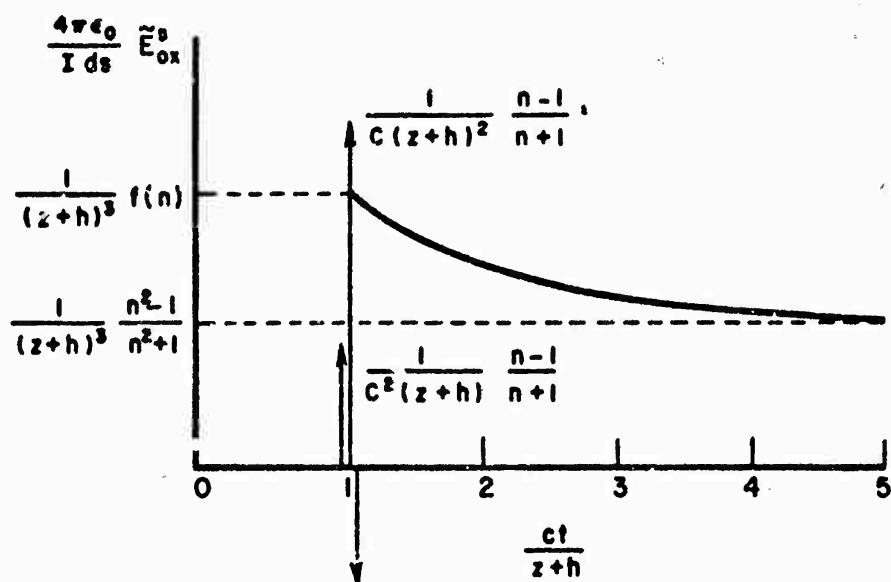$$\frac{ct}{z+h}$$

0   1   2   3   4   5

Fig. 5. Sketch of transient field of impulsively excited
horizontal electric dipole above half-space.
Observation point on vertical axis above
half-space.

The most significant difference between the interaction of vertical
and horizontal electric dipoles and a nondispersive half-space is the
addition of a doublet term, which varies as the inverse of twice the
dipole height above the half-space, for the horizontal dipole.

IV. SIMPLIFIED ESTIMATE OF RESULT FOR VERTICAL DIPOLE

It is evident from the complexity of the results for the time-depend-
ent fields and Hertz vectors presented in the first two sections that an
analytical utilization of the expressions would be extremely cumbersome.
However, sketches of the waveforms deduced from an examination of the
expressions are quite simple. This suggests that with the singularities
in the waveforms removed, very simple approximate expressions for the
waveform may be feasible. For a homogeneous half-space, the transient
fields for either vertical or horizontal dipoles must be quite simple in
form once the singularities associated with the interface are removed.

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Using the known initial and final values of the waveform for the vertical electric dipole in Eq. (3), an approximation of the form

$$(15) \qquad D(z, t) \simeq A + B\, e^{-\left(\frac{t - \tau}{\tau}\right)} \quad ,$$

where

$$A = \frac{2}{(z+h)^2} \left[\frac{4 n_h (n_v^2 - 1)}{(n_v(n_h+1))^2} + \frac{n_h - 1}{n_h + 1} - \frac{n_v n_h - 1}{n_v n_h + 1} \right] \quad ,$$

$$B = \frac{2}{(z+h)^2} \left[\frac{n_v n_h - 1}{n_v n_h + 1} \right] \quad ,$$

and

$$\tau = \frac{z+h}{c}$$

is suggested. In Eq. (15) the delay must be $(z+h)/c$, the time constant could in general be different but is set to $(z+h)/c$ here from observations of the exact waveform.

To test the approximation, the waveforms predicted by Eqs. (3) and (15) are compared in Figs. 6 and 7 for various refractive indices and both isotropic and anisotropic half-spaces. It is clear from these results that Eq. (15) is a reasonable approximation for refractive indices somewhat greater than $\sqrt{2}$ for both isotropic and anisotropic half-spaces. A transformation of Eq. (2) using Eq. (15) to replace Eq. (3) leads to a simple expression for the frequency-dependent field below a vertical dipole which should be a reasonable approximation over the entire spectrum. Admittedly, in most cases one is rarely interested in the fields directly below a vertical dipole; the value of these results in that sense lies in the demonstrated feasibility of using a simple time function to replace the complicated exact expression. They suggest that from a knowledge of the singularities and the initial and final values of the time-dependent waveform, simple estimates of the waveforms for more interesting observer locations may be possible. It will be shown later that the dipole-ground interactions obtained in the first two sections are two of the four interaction results needed for a theoretical model of the scattering from small ta   's located on the ground. Thus for the problem

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Fig. 6. Comparison of exact and approximate transient fields
of impulsively excited vertical electric dipole above
half-space. (a) $n_h = n_v = \sqrt{2}$; (b) $n_h = n_v = 2$;
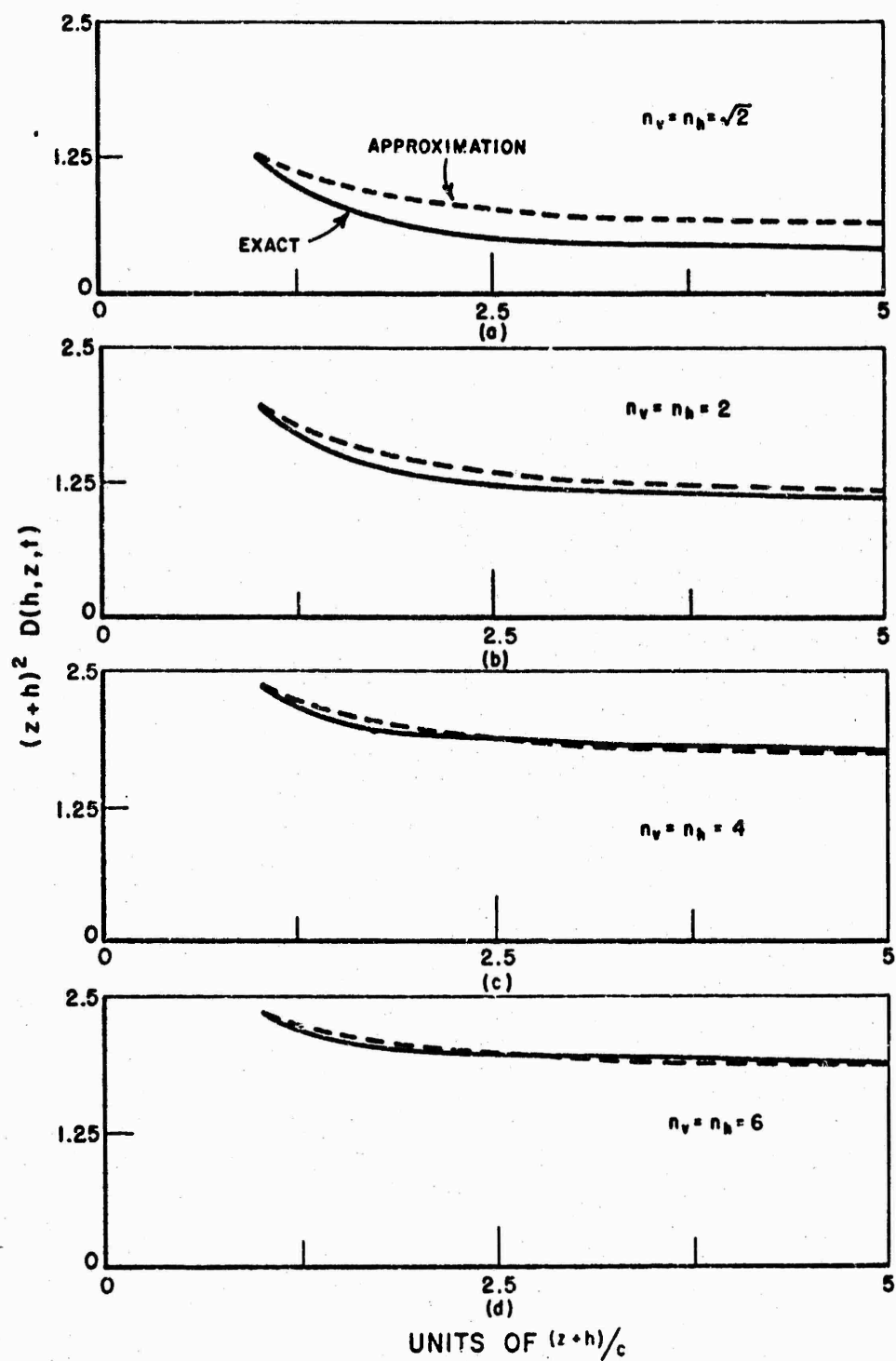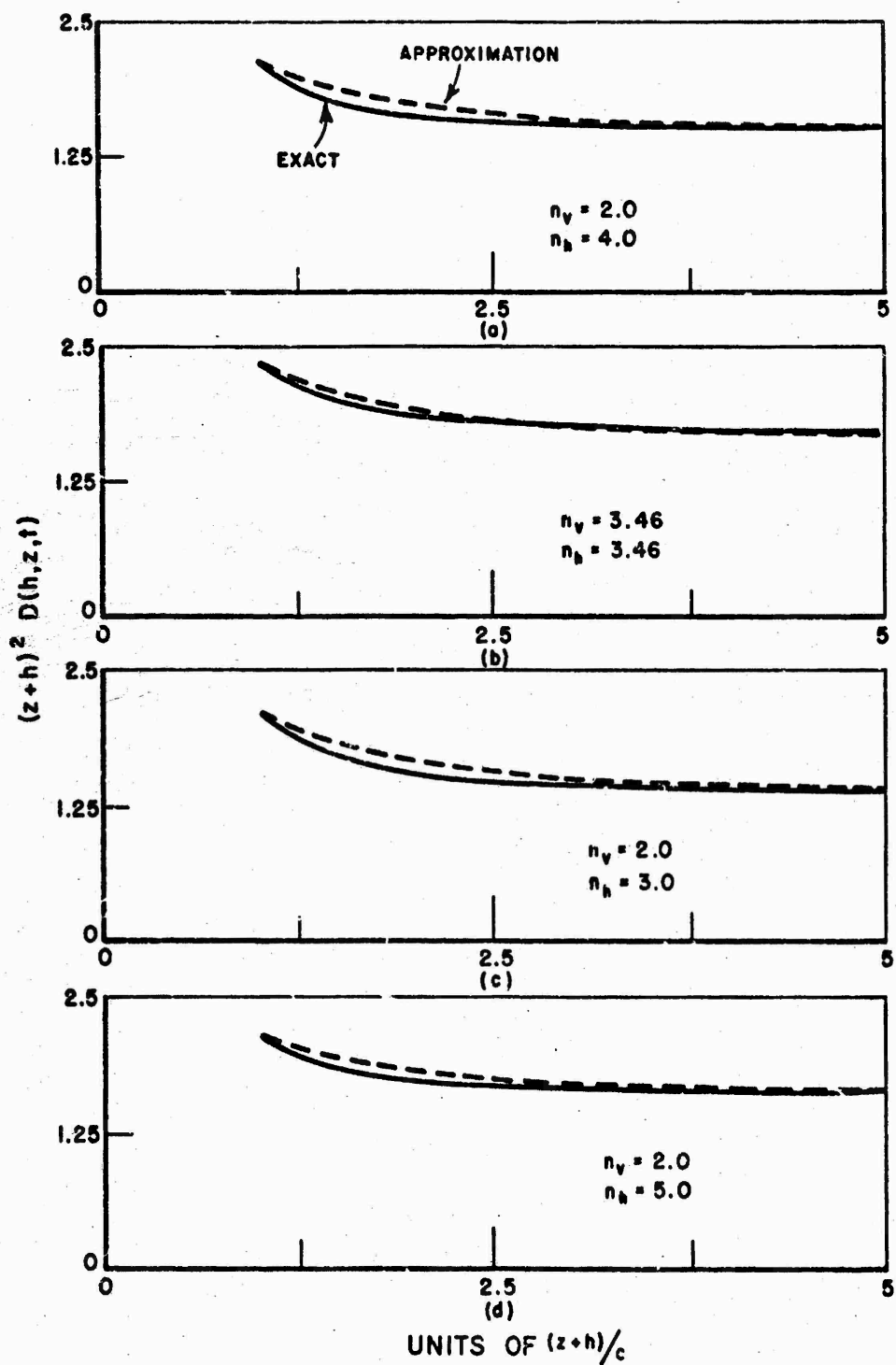(c) $n_h = n_v = 4$, (d) $n_h = n_v = 6$.

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Fig. 7. Comparison of exact and approximate transient fields of impulsively excited vertical electric dipole above half-space. (a) $n_h = 4$, $n_v = 2$; (b) $n_h = n_v = 3.46$; (c) $n_h = 3$, $n_v = 2$; (d) $n_h = 5$, $n_v = 2$.

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at hand, the transient fields on a vertical axis containing the dipole source, and approximations for them, are of more than academic interest.

V.   EFFECTS OF LOSS

If the half-space is dispersive or stratified, the techniques utilized in the previous sections are not useful for obtaining the transient fields. In these cases, even when the dielectric constant and conductivity of the half-space are assumed to be constant with frequency, an exact closed form solution for the transient fields does not appear to be possible. An exception is noted for a particular dispersive half-space,[6] rather unrelated to the ground effects of interest.

If the half-space dielectric constant and conductivity are assumed constant with frequency and attention is confined to sources and observation points above or on the half-space, then certain deductions concerning the form of the transient fields are possible. In the limit of long times (low frequencies) the fields must decay to their static values, and the long time values of the transient waveforms can be obtained from quasi-static estimates of the frequency-dependent fields and the final value theorem. A more sophisticated long time estimate (holding for shorter times) is obtained by assuming the displacement currents are negligible. That is, the propagation constant of the half-space is approximated as $k_1 = -j\omega\mu_0\sigma_1$. This approximation has been used in most of the analytical solutions for the transient fields of dipole sources in infinite and semi-infinite conducting media.[7]   For short times, estimates of the fields from high frequency asymptotic approximations are indicated. If the observation point and source dipole are confined to be above or on the half-space and only the secondary (fields due to the presence of the half-space) transient fields are considered, then these transient fields must be characterized by singularities and a jump discontinuity at the initial time of arrival of the field followed by a smooth decay to the final static value. That is, in general when the waveform first departs from zero singularities of various order and a jump discontinuity will occur, but beyond this initial arrival time the geometry of the problem is such that only a smooth decay of the waveform is possible. The above assumes the surface type fields, i.e., fields arriving partially via travel within the half-space can be neglected. For the asymptotic approximations of interest this means roughly that

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$$\tan\left(\frac{\rho}{h+z}\right) < \frac{\pi}{4} + \frac{1}{\sqrt{\epsilon_r}} + \frac{1}{4\epsilon_r} \quad ,$$

where ϵ_r is the relative dielectric constant of the half-space.

In this report, attention is confined to the final values of the transient fields and to the short time estimates predicted by high frequency asymptotic approximations. The asymptotic approximations utilized are those given by Baños.[8]

A. Long-Time Estimates

Bannister[9] has given quasi-static estimates of the frequency-dependent fields of vertical and horizontal dipoles above a conducting half-space. The geometry of the general problem is shown in Fig. 8. The long time values of the total transient fields for impulsive current excitation for vertical and horizontal electric dipoles at an arbitrary location above or on the half-space are given below.
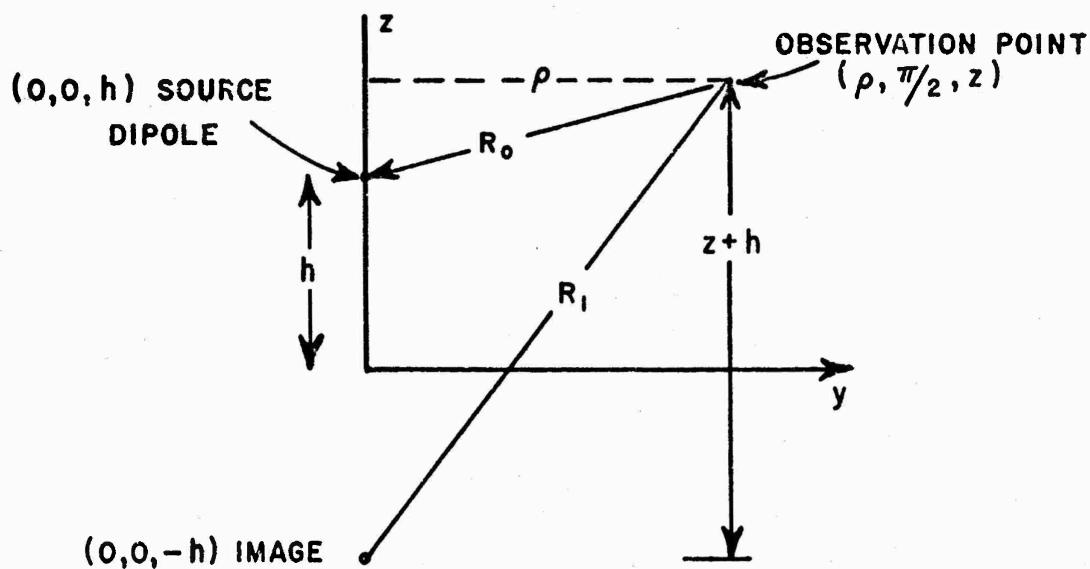

Fig. 8. Geometry of general problem.

Vertical Electric Dipole $(h \geq 0, \ z \geq 0)$

(16)
$$\tilde{E}_z(\rho, z, \infty) = - \frac{Ids}{4\pi\epsilon_0} \left[\frac{1}{R_0{}^3} \left(1 - \frac{3(z-h)^2}{R_0{}^2} \right) + \frac{1}{R_1{}^3} \left(1 - \frac{3(z+h)^2}{R_1{}^2} \right) \right] \quad .$$

where

$$R_0{}^2 = \rho^2 + (z \ h)^2$$

$$R_1{}^2 = \rho^2 + (z+h)^2 \qquad .$$

(17)
$$\hat{E}_\rho(\rho, z, \infty) = \frac{Ids}{4\pi\epsilon_0} \left[\frac{3\rho(z-h)}{R_0{}^5} + \frac{3\rho(z+h)}{R_1{}^5} \right]$$

(18)
$$\tilde{H}_\varphi(\rho, z, \infty) \equiv 0 \quad .$$

Horizontal Electric Dipole $(\rho \geq z+h, \ z \geq 0, \ h \geq 0)$

(19)
$$\tilde{E}_\rho(\rho, \phi, z, \infty) = \frac{Ids \cos\phi}{4\pi\epsilon_0} \left[\frac{3\rho^2}{R_0{}^5} - \frac{1}{R_0{}^3} - \frac{3\rho^2}{R_1{}^5} + \frac{1}{R_1{}^3} \right] \quad .$$

(20)
$$\tilde{E}_\phi(\rho, \phi, z, \infty) = \frac{Ids \sin\phi}{4\pi\epsilon_0} \left[\frac{1}{R_0{}^3} - \frac{1}{R_1{}^3} \right] \quad .$$

(21)
$$\tilde{E}_z(\rho, \phi, z, \infty) = \frac{Ids \cos\phi}{4\pi\epsilon_0} \left[\frac{3\rho(z-h)}{R_0{}^5} - \frac{3\rho(z+h)}{R_1{}^5} \right] \quad .$$

(22)
$$\tilde{H}_\rho(\rho, \phi, z, \infty) = \tilde{H}_\varphi(\rho, \phi, z, \infty) = \tilde{H}_z(\rho, \phi, z, \infty) = 0 \quad .$$

Note that the above expressions contain both the primary and scattered fields. A comparison of the long time values obtained here with those for a nondispersive half-space illustrates that if the half-space has

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some conductivity the final values of the transients are independent of the half-space parameters. Similar results, i.e., long time solutions, for vertical and horizontal magnetic dipoles can be obtained.

B. Short Time Estimates

Asymptotic approximations of the integral expressions resulting from an exact solution of the dipole, half-space problem are somewhat more difficult to obtain than those for the quasi-static limit. One also finds that such approximations are usually specialized to various regions. Baños,[8] for example, obtained three term asymptotic approximations in the vicinity of the vertical axis and in the vicinity of the surface interface but only two term expansions over the entire upper hemisphere.

1. Vertical axis

Baños[8] gives asymptotic approximations in the vicinity of the vertical axis which reduce to the following forms for observation points on the vertical axis ($\rho = 0$) and on or above ($z \geq 0$) the conducting half-space.

Vertical Electric Dipole ($\rho = 0$, $z \geq 0$)

$$(23) \quad E_z^s(0, z) = \frac{Z_0 \, Ids}{2\pi} \, e^{-j\frac{\omega}{c}(h+z)} \left\{ \frac{1}{(h+z)^2} \left[\frac{1-n}{1+n} \right] + \frac{1}{j\frac{\omega}{c}(h+z)^3} \left[\frac{3n-4n^2+1}{1+n} \right] \right.$$

$$\left. - \frac{6}{k_1 \frac{\omega}{c}(h+z)^4} \left[\frac{-1+3n-5n^2+3n^3}{1+n} \right] + \frac{j \, 12 \, E_2}{\left(\frac{\omega}{c}\right)^2 k_1(h+z)^5} \frac{1}{(1+n)} \right\} ,$$

where Z_0 is the free space impedance,

$$n = \frac{k_0}{k_1} \, \omega \left[\omega^2 \, \epsilon_r - j \, \frac{\omega \sigma_A}{\epsilon_0} \right]^{-\frac{1}{2}} \quad *$$

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* Note change in definition from that used in first two sections.

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$$E_z = -2 + 4n - 5n^2 + 3n^3 \quad ,$$

and ϵ_r, σ are the relative dielectric constant and conductivity of the half-space respectively. The above result is for

$$(24) \qquad \left| \sqrt{\left(\frac{\omega}{c}\right)^2 \epsilon_r - j\omega\mu_0\sigma_1(z+h)} \right| > 1.0 \quad .$$

For a dielectric half-space ($\sigma_1 = 0$) Eq. (23) yields a transient waveform

$$(25) \qquad \tilde{E}_z(z, t') = \frac{Z_0\,Ids}{2\pi(h+z)^2}\left\{ \frac{\sqrt{\epsilon_r}-1}{\sqrt{\epsilon_r}+1}\,\delta(t') + \left[\frac{c}{h+z}\left(\frac{3 - 4/\sqrt{\epsilon_r} + \sqrt{\epsilon_r}}{\sqrt{\epsilon_r}+1}\right)\right.\right.$$

$$+ \frac{6c^2 t'}{\sqrt{\epsilon_r}(h+z)^2}\left(\frac{\frac{3}{\epsilon_r} - \frac{5}{\sqrt{\epsilon_r}} + 3 - \sqrt{\epsilon_r}}{\sqrt{\epsilon_r}+1}\right) + \frac{6c^3 t'^2}{\sqrt{\epsilon_r}(h+z)^3}$$

$$\left.\left.\cdot \left(\frac{\frac{3}{\epsilon_r} - \frac{5}{\sqrt{\epsilon_r}} + 4 - 2\sqrt{\epsilon_r}}{\sqrt{\epsilon_r}+1}\right)\right] u(t')\right\} \quad ,$$

where $t' = t - (z+h/c)$.

It can be shown that the latter result agrees with Eq. (2) for t' equal zero. That is, for a dielectric half-space both the delta function singularity and jump discontinuity are correctly predicted by the asymptotic approximation.

Horizontal Electric Dipole

For the horizontal electric dipole, Baños[8] gives an asymptotic expansion for observation points above or on the half-space and in the vicinity of the vertical axis for the condition in Eq. (24). On the vertical axis, this result reduces to

(26) $E_x^s(z) = \dfrac{Z_0 \, Ids}{2\pi} \left\{ \dfrac{j \frac{\omega}{c}}{2(z+h)} \left(\dfrac{1-n}{1+n}\right) + \dfrac{1}{.2(z+h)^2} \left(\dfrac{1-n}{1+n}\right) \right.$

$\left. + \dfrac{1}{j \frac{2\omega}{c}(z+h)^3} \left[\dfrac{2n^4 - 6n^3 + 2n^2 + n + 1}{1+n}\right] \right\} e^{-j \frac{\omega}{c}(h+z)} .$

For a dielectric half-space $(\sigma_1 = 0)$, Eq. (26) yields a transient waveform

(27) $\tilde{E}_x^s(z, t) = \dfrac{Z_0 \, Ids}{2\pi(z+h)} \left\{ \dfrac{1}{2c} \left(\dfrac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1}\right) \delta'(t') + \dfrac{1}{2(z+h)} \left(\dfrac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1}\right) \delta(t') \right.$

$\left. + \dfrac{c}{2(z+h)^2(\sqrt{\epsilon_r}+1)} \left[\dfrac{\epsilon_r^2 + \epsilon_r^{3/2} + 2\epsilon_r - 6\sqrt{\epsilon_r} + 2}{\epsilon_r^{3/2}}\right] u(t') \right\} .$

Note that the first two terms in Eq. (27), i.e., the doublet and impulse weights, agree with those in Eq. (12). However, the coefficient of the step discontinuity in Eq. (27) does not agree with that obtained from Eq. (12)[11] when the differentiations are carried out. It is shown in Reference 11 that the coefficient from Eq. (12) is

(28) $\dfrac{c \, Z_0 \, Ids}{2(z+h)^2 2\pi} \left\{ \dfrac{(\sqrt{\epsilon_r}-1)(\sqrt{\epsilon_r}+2)}{\sqrt{\epsilon_r}(\sqrt{\epsilon_r}+1)} + \dfrac{2(\sqrt{\epsilon_r}-1)^2}{\epsilon_r^2(\sqrt{\epsilon_r}+1)^3} [2\epsilon_r^2 + 6\epsilon_r^{3/2} + 5\epsilon_r + \sqrt{\epsilon_r} + 2] \right\} .$

If the half-space is dispersive, the long-time values of the transient fields given in Eqs. (16) - (22) hold for any observation point in the upper hemisphere $z \geq 0$. For observation points on the vertical axis in the upper hemisphere, some properties of the short-time behaviour can be obtained from the asymptotic results in Eqs. (23) and (26). For the vertical electric dipole, the weight of the impulse singularity at t' equal to zero is

(29) $f_1(z, 0) = \dfrac{Z_0 \, Ids}{2\pi(h+z)^2} \left[\dfrac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1}\right] ,$

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and the step discontinuity is

$$(30) \qquad f_2(z, 0) = \frac{Z_0 \, Ids}{2\pi(h+z)^2(1+\sqrt{\epsilon_r})} \left\{ \frac{\sigma}{\epsilon_0 \sqrt{\epsilon_r}(1+\sqrt{\epsilon_r})} + \frac{c}{h+z} \left[3 - \frac{4}{\sqrt{\epsilon_r}} + \sqrt{\epsilon_r} \right] \right\} .$$

For the horizontal electric dipole, similar utilization of the initial value theorem yields

$$(31) \qquad f_0(z, 0) = \frac{Z_0 \, Ids}{2\pi} \cdot \frac{1}{2c(z+h)} \left[\frac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1} \right] ,$$

for the weight of the doublet term and

$$(32) \qquad f_1(z, 0) = \frac{Z_0 \, Ids}{2\pi} \left\{ \frac{\sigma}{\epsilon_0 \sqrt{\epsilon_r}(\sqrt{\epsilon_r} + 1)} \cdot \frac{1}{2c(z+h)} + \frac{1}{2(z+h)^2} \left[\frac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1} \right] \right\} ,$$

for the weight of the impulse term. Note that for both the vertical and horizontal dipoles, the weight of the highest order singularity is identical to that for the corresponding dielectric half-space regardless of the conductivity of the half-space. For the horizontal dipole, the magnitude of the jump discontinuity is not obtained.

To establish an estimate of the transient waveforms predicted by the asymptotic estimates in Eqs. (23) and (26) a Fourier synthesis technique,[2] i.e., a numerical inversion of the frequency-dependent expressions using calculations at harmonically related frequencies, can be used. In using this technique, the period of the fundamental frequency is selected to be at least 10 times the relaxation time ($T_r = \epsilon/\sigma$), and at the same time the relative parameters of the half-space and the dipole height are selected to satisfy Eq. (24). In Fig. 9, the normalized background or backscattered response waveforms for a vertical electric dipole for a height $z = h = 200$ meters, half-space relative dielectric constant of 12 and conductivities of 0.1, 0.01 and 0.001 mhos per meter are shown. The abscissa scale is in units of the relaxation time of the half-space, and the coefficient of the impulse term (Eq. (29)) has been subtracted from the calculations before synthesis. Corresponding waveforms for the horizontal dipole for the same parameters are shown in Fig. 10. Note that in this case the coefficient of the doublet term (Eq. (31)) has been subtracted and also a reflection coefficient type expression

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Fig. 9. Transient backscattered fields of impulsively excited
vertical electric dipole above conducting half-space.
$\epsilon_r = 12$, h = 200 m, $\sigma = 0.1$, 0.01, 0.001. Impulse
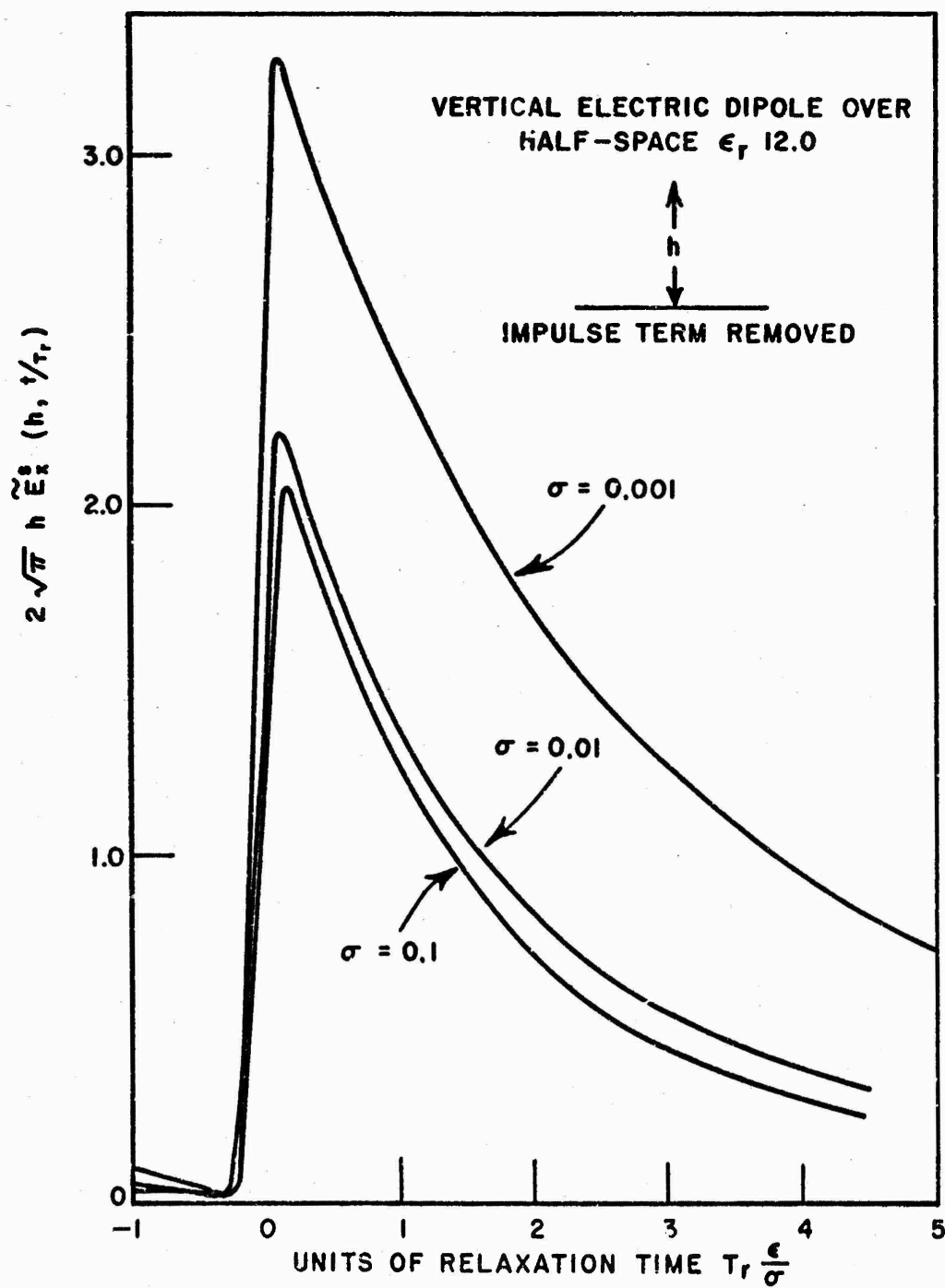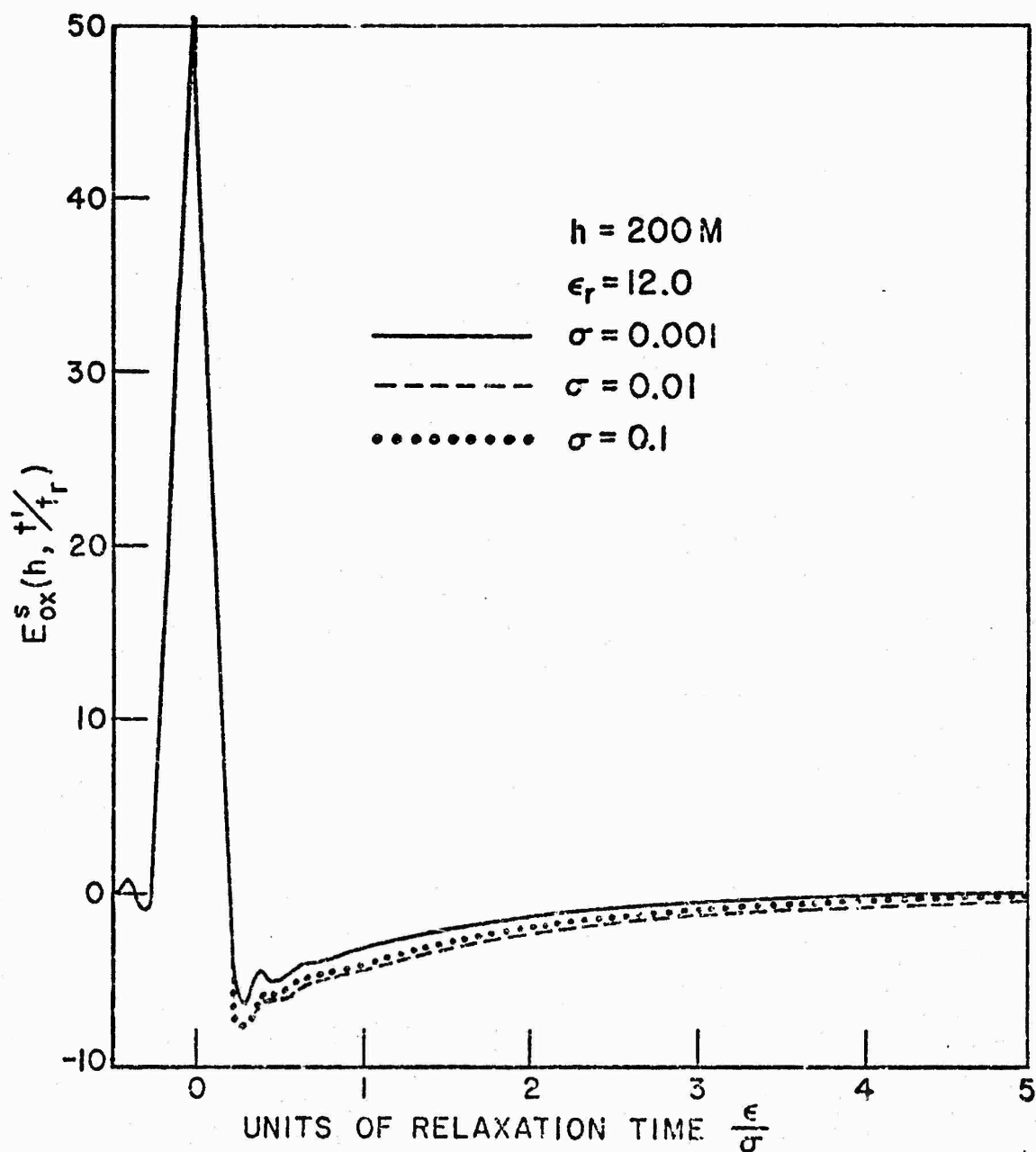singularity removed.

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Fig. 10. Transient backscattered fields of impulsively excited horizontal electric dipole above conducting half-space. $\epsilon_r = 12$, h = 200 m, σ = 0.1, 0.01, 0.001. Doublet singularity and portion of impulse singularity removed.

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$$(33) \qquad f = \frac{1}{2(z+h)^2}\left(\frac{\sqrt{\epsilon_r}-1}{\sqrt{\epsilon_r}+1}\right) \qquad ,$$

has been subtracted before synthesis. It is evident from Fig. 10 that the impulse singularity weight is not independent of the half-space conductivity since the waveforms are still dominated by an impulse type singularity. An estimate of the magnitude of the jump discontinuity could be obtained by subtracting both Eqs. (31) and (32) before synthesis. However, it is clear from Fig. 9 and Eq. (30) that the synthesis procedure cannot faithfully estimate the discontinuities involved for conductivities in this range except possibly by using many more harmonics than the 40 used for the results in Figs. 9 and 10. For either the vertical or horizontal dipoles, the essential form of the transient waveforms is clear but an extensive computational program would be required to establish exact details of the waveforms over a range of half-space parameters, source dipole heights and observer locations. Summarized below are the established initial and final values of the transient waveforms for both vertical and horizontal dipoles for observation points on the vertical axes and in the upper hemisphere ($z \geq 0$). These values are for the z component of the electric field for the vertical electric dipole and horizontal component of the electric field parallel to the dipole orientation for the horizontal dipole.

Vertical Electric Dipole

Half-space	Initial Value (jump discontinuity)	Final Value
Perfect Conductor	$\dfrac{Ids}{2\pi\epsilon_0(z+h)^3}$	$\dfrac{Ids}{2\pi\epsilon_0(z+h)^3}$
Lossless Dielectric	$\dfrac{Ids}{2\pi\epsilon_0(z+h)^3}\left[\dfrac{4(\sqrt{\epsilon_r}-1)}{\sqrt{\epsilon_r}(1+\sqrt{\epsilon_r})}-\dfrac{\sqrt{\epsilon_r}-1}{\sqrt{\epsilon_r}+1}\right]$	$\dfrac{Ids}{2\pi\epsilon_0(z+h)^3}\left[\dfrac{\epsilon_r-1}{\epsilon_r+1}\right]$

Conducting
$$\frac{Ids}{2\pi\epsilon_0(h+z)^2}\left\{\frac{Z_0\sigma}{\sqrt{\epsilon_r}(\sqrt{\epsilon_r}+1)^2}\right.$$
$$\left.+\frac{1}{h+z}\left[\frac{4(\sqrt{\epsilon_r}-1)}{\sqrt{\epsilon_r}(1+\sqrt{\epsilon_r})}+\frac{\sqrt{\epsilon_r}-1}{\sqrt{\epsilon_r}+1}\right]\right\}\frac{Ids}{2\pi\epsilon_0(z+h)^3}$$

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Horizontal Electric Dipole

	Initial Value	Final Value

Perfect Conductor $\dfrac{Ids}{4\pi\epsilon_0(z+h)^3}$ $\dfrac{Ids}{4\pi\epsilon_0(z+h)^3}$

Lossless Dielectric $\dfrac{Ids}{2\pi\epsilon_0}\dfrac{c}{2(z+h)^2}\left\{\dfrac{(\sqrt{\epsilon_r}-1)(\sqrt{\epsilon_r}+1)^2}{\sqrt{\epsilon_r}(\sqrt{\epsilon_r}+1)}\right.$

$$+\frac{2(\sqrt{\epsilon_r}-1)^2}{\sqrt{\epsilon_r}^2(\sqrt{\epsilon_r}+1)^2}\cdot$$

$$\left.\cdot\left[2\epsilon_r^2+6\epsilon_r^{3/2}+5\epsilon_r+\sqrt{\epsilon_r}+2\right]\right\}\qquad \dfrac{Ids}{4\pi\epsilon_0(z+h)^3}\left(\dfrac{\epsilon_r-1}{\epsilon_r+1}\right)$$

Conducting ------------------------ $\dfrac{Ids}{4\pi\epsilon_0(z+h)^3}$

2. Entire upper hemisphere $(z \geq 0)$

Baños[8] gives two term asymptotic expansions for observation points anywhere in the upper hemisphere $z \geq 0$.

Vertical Electric Dipole

$$(34)\qquad E_z^s(\theta)=\frac{Ids\,Z_0}{4\pi}\left\{\left(j\frac{k_0}{R_1}+\frac{1}{R_1^2}-j\frac{[k_0^2(h+z)^2+1]}{k_0R_1^3}-\frac{3(h+z)^2}{R_1^4}+j\frac{3(h+z)^2}{k_0R_1^5}\right.\right.$$

$$+\frac{2\cos\theta}{\cos\theta+n\sqrt{1-n^2\sin^2\theta}}\left[-j\frac{k_0}{R_1}-\frac{1}{R_1^2}+j\frac{(k_0^2(h+z)^2+1)}{k_0R_1^3}+\frac{3(h+z)^2}{R_1^4}\right.$$

$$-j\frac{3(h+z)^2}{k_0R_1^5}+\frac{E_1}{R_1^2}+E_1\left\{\frac{-j}{k_0R_1^3}-\frac{(k_0^2(h+z)^2+2)}{k_0^2R_1^4}+j\frac{5(h+z)^2}{k_0R_1^5}\right.$$

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(34)
(cont.)

$$+ \frac{8(h+z)^2}{k_o^2 R_1^6} \Bigg] \Bigg] \Bigg) e^{-jk_o R_1} + \frac{2 e^{-jk_1 \rho} e^{-k_1(h+z)\sqrt{1-n^2}}}{\rho^2 \left(1 - \frac{j \cot\theta}{\sqrt{1-n^2}}\right)^{3/2}} -$$

$$\left(\frac{n}{1-n^2}\right) u(\theta - \theta_o) \Bigg\} \quad ,$$

where

$$h + z = R_1 \cos\theta \ , \qquad \rho = R_1 \sin\theta$$

$$\theta_o = \frac{\pi}{4} + |n| + \frac{1}{4}|n|^2 + O|n|^4$$

$$E_1 = -1 + \frac{\sin^2\theta}{1 - n^2 \sin^2\theta} \left[\frac{n^3 \cos\theta + \sqrt{1 - n^2 \sin^2\theta}}{\cos\theta + n\sqrt{1 - n^2 \sin^2\theta}}\right]^2$$

$$+ \frac{\cos\theta + \frac{n^3}{2}\left(\frac{2\cos^2\theta - 3\sin^2\theta}{\sqrt{1 - n^2 \sin^2\theta}}\right) + \frac{n^5}{2} \frac{\sin^2\theta \cos^2\theta}{(1 - n^2 \sin^2\theta)^{3/2}} - \frac{\sin^2\theta}{\cos\theta}}{\cos\theta + n\sqrt{1 - n^2 \sin^2\theta}}$$

For a dielectric half-space, the asymptotic approximation in Eq. (34) yields a transient waveform

$$(35) \qquad \tilde{E}_z^s(\theta, t) = \frac{Ids\, Z_0}{4\pi} \Bigg\{ \left[1 - \left(\frac{h+z}{R_1}\right)^2 + \frac{2\,\epsilon_r \cos\theta}{\epsilon_r \cos\theta + \epsilon_r - \sin^2\theta}\left(\left(\frac{h+z}{R_1}\right)^2 - 1\right)\right]$$

$$\frac{\delta'\left(t - \frac{R_1}{c}\right)}{cR_1} + \left[1 - 3\left(\frac{h+z}{R_1}\right)^2 + \frac{2\epsilon_r \cos\theta}{\epsilon_r \cos\theta + \sqrt{\epsilon_r - \sin^2\theta}}\right.$$

$$\left(3\left(\frac{h+z}{R_1}\right)^2 - 1 + E_1\left(1 - \left(\frac{h+z}{R_1}\right)^2\right)\right)\Bigg] \frac{\delta\left(t - \frac{R_1}{c}\right)}{R_1^2}$$

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$$(35) \atop (\text{cont.}) \qquad + \left[1 - 3\left(\frac{h+z}{R_1}\right)^2 + \frac{2\,\epsilon_r \cos\theta}{\epsilon_r \cos\theta + \sqrt{\epsilon_r - \sin^2\theta}} \left(3\left(\frac{h+z}{R_1}\right)^2 - 1 \right. \right.$$

$$\left. \left. + E_1 \left(1 - 5\left(\frac{h+z}{R_1}\right)^2 \right) \right) \right] \frac{c\,u\left(t - \frac{R_1}{c}\right)}{R_1^3}$$

$$+ \frac{4\,c^2\,\epsilon_r \cos\theta\, E_1}{(\epsilon_r \cos\theta + \sqrt{\epsilon_r - \sin^2\theta})R_1^4} \left(t - \frac{R_1}{c} \right) \left[1 - 4\left(\frac{h+z}{R_1}\right)^2 \right] u\left(t - \frac{R_1}{c} \right)$$

$$+ \frac{2\sqrt{\epsilon_r}\; u(\theta - \theta_0)}{\pi\sqrt{\rho}(\epsilon_r - 1)^{1/4}(\epsilon_r R_1^2 - \rho^2)^{3/4}} \quad\cdot$$

$$\cdot \left\{ \left[\frac{\cos\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r - 1}}\right)\right]\left(\frac{h+z}{c}\right)\sqrt{\epsilon_r - 1} - \sin\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r - 1}}\right)\right]\left(\frac{\sqrt{\epsilon_r}\rho}{c} - t\right)}{\left(\frac{h+z}{c}\right)^2(\epsilon_r - 1) + \left(\frac{\epsilon_r \rho}{c} - t\right)^2} \right] \right\} ;$$

where in E_1 and θ_0, $n \to 1/\sqrt{\epsilon_r}$. The last term in Eq. (34) is inverse transformed under the condition that the time function be real. This procedure properly yields the delta function singularity at $t = \sqrt{\epsilon_r}\rho/c$ when $h = z = 0$ as is evident from Eq. (34) if we set $h = z = 0$ and $\theta = \pi/2$ before transforming. The waveform in Eq. (35) specialized either to the vertical axis ($\rho = 0$) or to the case of both source and observation point on the interface ($h = z = 0$) correctly predicts the singularities known from the exact solutions, but yields incorrect magnitudes for the jump discontinuities.

It is interesting to observe the precursor for the surface field (last term in Eq. (35)) when either the dipole source or observation point is raised above the interface and $\theta > \theta_0$. Similar effects have been noted[10] from asymptotic scattering approximations when the assumed path is not a minimum time path. In this case, the delay time $\sqrt{\epsilon_r}\rho/c$ is the maximum delay possible for a wave traveling via the dielectric. When shorter delays are possible, i.e., z or $h \neq 0$, the asymptotic approximation properly predicts a surface-type field prior to $\sqrt{\epsilon_r}\rho/c$. Previous experience indicates that such an estimate is good only in the vicinity of this time however.

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Horizontal Electric Dipole

For the horizontal electric dipole, the two term expansion by Banos[8] yields

(36)
$$E_{ox}^{s}(\theta) = \frac{Ids\,Z_o}{4\pi} \left\{ \left(j\,\frac{k_o}{R_1} + \frac{1}{R_1^2} + \frac{(1+k_o^2\rho^2)}{jk_oR_1^3} - \frac{3\rho^2}{R_1^4} - \frac{3\rho^2}{jk_oR_1^5} \right. \right.$$

$$- \frac{j2nk_o\cos\theta}{R_1[n\cos\theta + \sqrt{1-n^2\sin^2\theta}]} \left\{ 1 + \frac{E_1'}{-jk_oR_1} \right\}$$

$$+ \frac{2n^2\cos\theta}{\cos\theta + n\sqrt{1-n^2\sin^2\theta}} \left\{ -\frac{1}{R_1^2} - \frac{(1+k_o^2\rho^2)}{jk_oR_1^3} + \frac{3\rho^2}{R_1^4} + \frac{3\rho^2}{jk_oR_1^5} \right.$$

$$\left. + E_1\left(-\frac{(k_o^2\rho^2+2)}{k_o^2R_1^4} + \frac{1}{jk_oR_1^3} + j\,\frac{5\rho^2}{k_oR_1^5} + \frac{3\rho^2}{k_o^2R_1^6} \right) \right\} \right) e^{-jk_oR_1}$$

$$+ \frac{2n\,e^{-jk_1\rho}\,e^{-k_1(h+z)\sqrt{1-n^2}}}{\rho^2(1-n^2)\left[1 - j\,\dfrac{\cot\theta}{\sqrt{1-n^2}}\right]^{3/2}} \; u(\theta - \theta_o)$$

$$\left. + \frac{2n^4\,e^{-k_1(h+z)\sqrt{1-n^2}}\,e^{-jk_1\rho}}{k_ok_1(1-n^2)\left[1 - j\,\dfrac{\cot\theta}{\sqrt{1-n^2}}\right]^{3/2}} \left(-\frac{k_1^2}{\rho^2} + j\,\frac{4k_1}{\rho^3} - \frac{6}{\rho^4} \right) u(\theta - \theta_o) \right\} \quad ,$$

where

$$E_1' = -\frac{13}{8} + \frac{3}{8}\cot^2\theta + \frac{1}{8}\csc^2\theta + \frac{\sin^2\theta}{1-n^2\sin^2\theta}\left[\frac{n^2\cos\theta + n\sqrt{1-n^2\sin^2\theta}}{n\cos\theta + \sqrt{1-n^2\sin^2\theta}}\right]^2$$

$$+ \frac{n\cos\theta(1-n^2\sin^2\theta)^{3/2} + n^2\cos2\theta(1-n^2\sin^2\theta) - \frac{1}{4}n^4\sin^2 2\theta}{2(1-n^2\sin^2\theta)^{3/2}[n\cos\theta + \sqrt{1-n^2\sin^2\theta}]}$$

$$- \cot2\theta\cot\theta + \left(2\cot2\theta - \frac{\cot}{2}\right)\frac{\sin\theta[n^2\cos\theta + n\sqrt{1-n^2\sin^2\theta}}{\sqrt{1-n^2\sin^2\theta}[n\cos\theta + \sqrt{1-n^2\sin^2\theta}]}$$

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For a dielectric half-space, the asymptotic estimate in Eq. (36) yields a transient waveform

$$
\begin{aligned}
(37)\quad \hat{E}_{ox}^{s}(\theta,t) ={}& \frac{Ids\,Z_0}{4\pi}\left\{\left[1-\left(\frac{\rho}{R_1}\right)^2-\frac{2\cos\theta}{\cos\theta+\sqrt{\epsilon_r-\sin^2\theta}}+\frac{2\cos\theta}{\epsilon_r\cos\theta+\sqrt{\epsilon_r-\sin^2\theta}}\left(\frac{\rho}{R_1}\right)^2\right]\right.\\[2mm]
&\frac{\delta'\left(t-\frac{R_1}{c}\right)}{cR_1}+\left[1-3\left(\frac{\rho}{R_1}\right)^2+\frac{2\cos\rho\,\bar{E}_1{}'}{\cos\theta+\sqrt{\epsilon_r-\sin^2\theta}}-\frac{2\cos\theta}{\epsilon_r\cos\theta+\sqrt{\epsilon_r-\sin^2\theta}}\right.\\[2mm]
&\left.\cdot\left(1-3\left(\frac{\rho}{R_1}\right)^2+E_1\left(\frac{\rho}{R_1}\right)^2\right)\right]\frac{\delta\left(t-\frac{R_1}{c}\right)}{R_1{}^2}\\[2mm]
&+\left[1-3\left(\frac{\rho}{R_1}\right)^2+\frac{2\cos\theta}{\epsilon_r\cos\theta+\sqrt{\epsilon_r\sin^2\theta}}\left(-1+3\left(\frac{\rho}{R_1}\right)^2+E_1\left(1-5\left(\frac{\rho}{R_1}\right)^2\right)\right)\right]\\[2mm]
&\frac{cu\left(t-\frac{R_1}{c}\right)}{R_1{}^3}+2E_1{}'\left(1-4\left(\frac{\rho}{R_1}\right)^2\right)\frac{c^2\left(t-\frac{R_1}{c}\right)}{R_1{}^4}\;u\left(t-\frac{R_1}{c}\right)\\[2mm]
&+\frac{2(\epsilon_r-1)^{3/4}\,\mu(\theta-\theta_0)}{\pi\sqrt{\epsilon_r}\sqrt{\rho}\,(\epsilon_r R_1{}^2-\rho^2)^{3/4}}\\[2mm]
&\cdot\left[\frac{\cos\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r-1}}\right)\right]\left(\frac{h+z}{c}\right)\sqrt{\epsilon_r-1}-\sin\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r-1}}\right)\right]\left(\frac{\sqrt{\epsilon_r}\rho}{c}-t\right)}{\left(\frac{h+z}{c}\right)^2(\epsilon_r-1)+\left(\frac{\sqrt{\epsilon_r}\rho}{c}-t\right)^2}\right]\\[2mm]
&-\frac{2c\,u(\theta-\theta_0)}{\epsilon_r\rho^{3/2}(\epsilon_r-1)^{1/4}(\epsilon_r R_1{}^2-\rho^2)^{3/4}}\\[2mm]
&\cdot\int_{-\infty}^{\infty}\left[\frac{\cos\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r-1}}\right)\right]\left(\frac{h+z}{c}\right)\sqrt{\epsilon_r-1}-\sin\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r-1}}\right)\right]\left(\frac{\sqrt{\epsilon_r}\rho}{c}-\tau\right)}{\left(\frac{h+z}{c}\right)^2(\epsilon_r-1)+\left(\frac{\sqrt{\epsilon_r}\rho}{c}-\tau\right)^2}\right]\\
&\hspace{12cm}u(t-\tau)\,d\tau
\end{aligned}
$$

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(37)
(cont.)
$$+ \frac{12\, c^2\, u(\theta - \theta_0)}{\epsilon_r^{3/2}\, \rho^{5/2}\, (\epsilon_r - 1)^{1/4}\, (\epsilon_r R_1^2 - \rho^2)^{3/4}} \cdot$$

$$\cdot \int_{-\infty}^{\infty} \frac{\cos\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r-1}}\right)\right]\left(\frac{h+z}{c}\right)\sqrt{\epsilon_r-1} - \sin\left[\frac{3}{2}\tan^{-1}\left(\frac{-\sqrt{\epsilon_r}(h+z)}{\rho\sqrt{\epsilon_r-1}}\right)\right]\left(\frac{\sqrt{\epsilon_r}\rho}{c} - \tau\right)}{\left(\frac{h+z}{c}\right)^2 (\epsilon_r - 1) + \left(\frac{\sqrt{\epsilon_r}\rho}{c} - \tau\right)^2} \cdot$$

$$\cdot (t - \tau)u(t - \tau)\, d\tau \bigg\}$$

where in E_1', E_1 and θ_0, $n \to 1/\sqrt{\epsilon_r}$.

Again, specializing Eq. (37) to the vertical axis ($\theta = 0$, $\rho = 0$) one finds that the singularities agree with those in Eq. (27) but the jump magnitude does not agree with the result given in the Table. We conclude that the two term asymptotic expansions given by Baños[8] correctly predict only the highest order singularities in the transient waveforms for a dielectric half-space. It follows that the dependence of the jump discontinuity on the conductivity of the half-space for arbitrary observation points cannot be established from these results. A numerical inversion of Eqs. (34) and (36) with the singularities removed would be pointless since the results for a dielectric half-space are incorrect. Formally, one could derive a third term in the expansion using the procedure given by Baños. Assuming this led to the correct jump discontinuities for a dielectric half-space, dependence of the jump discontinuity on the conductivity might then be inferred with some confidence. For observation points where the last three terms of Eq. (36) are needed, i.e., $\theta > \theta_0$, the precursor seen in the case of the vertical dipole is again evident except when both the dipole and observation point lie on the interface.

For both the vertical and horizontal electric dipoles, application of the initial value theorem to Eqs. (34) or (36) yields the same weights for the doublet and impulse singularities as given in Eqs. (35) and (37) respectively. That is, the two term asymptotic expansions predict doublet and impulse singularities independent of the half-space conductivity. We have already seen that for the horizontal dipole, an impulse weight dependent on the half-space conductivity is obtained from a three term expansion on the vertical axis. As noted, on the vertical axis the two term expansions correctly predict the singularities for a dielectric half-space. However, for a dispersive half-space the two term expansions are only reliable in predicting the weight of the doublet singularity in the transient waveform. Unfortunately, the two term

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expansions given by Baños appear to be the most advanced asymptotic development available for observation points in the entire upper hemisphere. It seems clear that additional terms are needed in order to estimate the effects of the half-space conductivity on the lower order singularities and jump discontinuities of the transient waveform. Note that this specifically requires that the displacement currents not be neglected because it is the short time character of the waveform which is of interest. Thus the numerous developments where displacement currents are neglected[7] are not useful in resolving the singularities and discontinuities.

3. Vicinity of interface

Baños[8] has also given three term asymptotic expansions for the fields produced by both vertical and horizontal electric dipoles located above a conducting half-space for observation points in the vicinity of the interface. That is, it is assumed for these results that both the observer and source dipole heights (z and h respectively) are much less than the horizontal range, ρ, of the observation point. A major portion of the book is devoted to this development and the vertical axis results utilized earlier were intended as a supplement for small horizontal ranges where the interface results are not applicable.

Explicit expressions for the fields in the vicinity of the interface, for either vertical or horizontal dipole sources, are quite lengthy and will not be given here. Note that Bños does not present explicit expressions for the fields, his results are asymptotic expansions of certain basic integrals from which the fields are obtained using standard formulas[8] involving primarily second derivat' with respect to the observer height or horizontal range. No simpxification of the expressions occurs if either the source dipole or observer is located on the interface, and one notes the same precursor effects found with the two term upper hemisphere expansions. However, if both observer and source are on the interface, then the expressions for the fields are greatly simplified. Also, for this case we have an exact result for the vertical dipole which offers a check on the asymptotic development in the limit of zero conductivity.

For the vertical electric dipole, if both source and observer are on the interface, then

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(38)
$$E_z^s(\rho) = \frac{Z_0\,Ids}{4\pi}\left\{\left(j\,\frac{k_0}{\rho} + \frac{1}{\rho^2}\left[1 - \frac{2}{n^2(1-n^2)}\right]\right.\right.$$

$$+ \frac{1}{jk_0\rho^3}\left[1 - \frac{2}{n^2(1-n^2)} + \frac{3}{n^4(1-n^2)}\right] + \frac{2}{n^2(1-n^2)k_0^2\rho^4}\left(\frac{6}{n^2} + \frac{15}{n^4}\right)e^{-jk_0\rho}$$

$$\div\left(\frac{2n}{(1-n^2)\rho^2} - \frac{4}{j^{1}{}_0\rho^3(1-n^2)}\left[\frac{5n^2}{2} + 3n^4\right] - \frac{1}{\rho^4 k_0^2(1-n^2)}\right.$$

$$\left.\left.\frac{1}{\rho^4 k_0^2(1-n^2)}\left[\frac{39n^2}{2} + \frac{15n^2}{2(1-n^2)} + 12n^3 + 60n^5\right]\right)e^{-j\frac{k_0\rho}{n}}\right\}$$

where in the three term expansion for the integral related to the phase delay $k_0\rho/n$, certain terms have been deleted. It is possible to do this judiciously in the case because the exact solution for the transient fields for a dielectric half-space is known. For a dielectric half-space the result in Eq. (38) yields a waveform

(39)
$$\hat{E}_\rho^s(\rho, t) = \frac{Z_0\,Ids}{4\pi}\left\{\frac{\delta'\left(t - \frac{\rho}{c}\right)}{c\rho} + \frac{(\epsilon_r - 1 - 2\epsilon_r^2)}{(\epsilon_r - 1)\rho^2}\,\delta\left[t - \frac{\rho}{c}\right]\right.$$

$$+ \frac{c}{\rho^3}\left[1 + \frac{3\epsilon_r^3 - 2\epsilon_r^2}{\epsilon_r - 1}\right]u\left(t - \frac{\rho}{c}\right) + \frac{2\sqrt{\epsilon_r}}{(\epsilon_r - 1)\rho^2}\,\delta\left[t - \frac{\sqrt{\epsilon_r}\,\rho}{c}\right]$$

$$\left.- \frac{2c}{\rho^3}\left[\frac{5\epsilon_r + 6}{\epsilon_r(\epsilon_r - 1)}\right]u\left(t - \frac{\sqrt{\epsilon_r}\,\rho}{c}\right) + \dots\right\}$$

where the terms with a $1/\rho^4$ dependence have not been written out. It can be shown, by adding the direct term to Eq. (39), that the singularity and jump at $t = \rho/c$ and the singularity at $t = \sqrt{\epsilon_r}\,\rho/c$ agree with the exact solution in Eq. (10) for an isotropic half-space. It is interesting to note that no deletions were necessary to obtain agreement at $t = \rho/c$ and only one deletion was necessary to obtain agreement for the singularity at $t = \sqrt{\epsilon_r}\rho/c$. However, it was not possible by any inclusion or omission of

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terms to obtain agreement for the jump discontinuity at $t = \sqrt{\epsilon_r}\,\rho/c$. In the asymptotic expansions of the integrals related to the phase delays $k_o\rho$ and $k_o\rho/n$, terms to order $1/\rho^4$ were retained in each case.

For the horizontal electric dipole with both source and observer on the interface and retaining terms only to $1/\rho^3$

$$(40)\quad E_{ox}^s(\rho) = \frac{Z_o\,Ids}{4\pi}\left\{\left(-\frac{2}{\rho^2}\left[1+\frac{n^2}{1-n^2}\right] - \frac{2}{jk_o\rho^3}\left[1+\frac{n^2}{1-n^2}-\frac{\left(4-\frac{6}{n^2}\right)}{n^4(1-n^2)}\right]\right)\cdot e^{-jk_o\rho}\right.$$
$$\left. +\left(\frac{1}{\rho^2}\left[\frac{2n}{1-n^2}-\frac{2n^3}{1-n^2}\right]+\frac{1}{jk_o\rho^3}\left[\frac{n^2}{1-n^2}(-2+4n^2+6n^4)\right]\right)e^{-j\frac{k_o\rho}{n}}\right\}.$$

For a dielectric half-space, Eq. (40) yields a waveform

$$(41)\quad \tilde{E}_{ox}^s(\rho,t) = \frac{Z_o\,Ids}{4\pi}\left\{-\frac{2\epsilon_r}{(\epsilon_r-1)\rho^2}\ \delta\left[t-\frac{\rho}{c}\right]-\frac{2c}{\rho^3}\left(\frac{6\epsilon_r^4-4\epsilon_r^3+\epsilon_r}{\epsilon_r-1}\right)u\left[t-\frac{\rho}{c}\right]\right.$$
$$\left. +\frac{2}{\sqrt{\epsilon_r}\,\rho^2}\ \delta\left[t-\frac{\sqrt{\epsilon_r}\,\rho}{c}\right]+\frac{c(6+4\epsilon_r-2\epsilon_r^2)}{\epsilon_r^2\rho^3(\epsilon_r-1)}\ u\left[t-\frac{\sqrt{\epsilon_r}\,\rho}{c}\right]\right\}.$$

For a horizontal electric dipole, if both source and observer are on the interface ($h = z = 0$) and the half-space is nondispersive an exact closed form solution for the radial component of the transient field is easily[11] obtained as

$$(42)\quad \tilde{E}_\rho(\rho,t) = \frac{Ids\,\cos\phi\,Z_o}{2\pi\rho^2}\left\{-\frac{1}{\epsilon_r-1}\ \delta\left[t-\frac{\rho}{c}\right]+\frac{\epsilon_r^{3/2}-\epsilon_r^{1/2}-1}{\epsilon_r^{1\,2}(\epsilon_r-1)}\ \delta\left[t-\frac{\sqrt{\epsilon_r}\,\rho}{c}\right]\right.$$
$$+\frac{\epsilon_r}{\epsilon_r-1}\left[3(\epsilon_r+1)^2\left(\frac{ct}{\rho T}\right)^5+5(\epsilon_r+1)\left(\frac{ct}{\rho T}\right)^3-2\frac{c}{\rho T}+\frac{(\epsilon_r-1)c}{\epsilon_r}\frac{c}{\rho}\right]\cdot$$
$$\left. \cdot\left[u\left[t-\frac{\rho}{c}\right]-u\left[t-\frac{\sqrt{\epsilon_r}\,\rho}{c}\right]\right]+\frac{2c}{(\epsilon_r+1)\rho}\ u\left[t-\frac{\sqrt{\epsilon_r}\,\rho}{c}\right]\right\},$$

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where

$$T = \left[(\epsilon_r + 1) \left(\frac{ct}{\rho} \right)^2 - \epsilon_r \right]^{1/2} \quad .$$

For the same configuration and a conducting half-space, Wait[7] has given an approximate solution for the component of the radial field in the xz plane ($\phi = 0$), neglecting displacement currents, as

$$(43) \qquad \tilde{E}_x(\rho, t) = \frac{Ids}{2\pi \sigma \rho^3} \left\{ \delta(t) + \frac{(\sigma\mu_o)^{3/2} \rho^3}{4\sqrt{\pi} t^{5/2}} e^{-\frac{\sigma\mu_o\rho^2}{4t}} u(t) \right\} \quad .$$

The results in Eqs. (42) and (43) represent the extremes, respectively, of neglecting the conduction and displacement currents. From Eq. (40), the weight of the delta function singularity for the seconary field is

$$(44) \qquad f_1(\rho, t) = - \frac{Ids\, Z_o \epsilon_r}{2\pi(\epsilon_r - 1)} \quad ,$$

and the magnitude of the jump discontinuity is

$$(45) \qquad f_2(\rho, t) = \frac{Ids\, Z_o}{4\pi} \left\{ \frac{\sigma}{\epsilon_o(\epsilon_r - 1)\rho^2} + \frac{2c}{\rho^3} \left[\frac{4\epsilon_r^3 - 7\epsilon_r}{\epsilon_r - 1} \right] \right\} \quad .$$

From the results in Eqs. (43), (44) and (45), a reasonable estimate of the radial field for an impulsive current excitation for all times should be feasible. Wait,[7] has plotted Eq. (43) (second term) and this result could be corrected for short times using Eqs. (44) and (45).

VI. SCATTERING MODEL FOR TARGETS LOCATED ON A HALF-SPACE

A. Effect of Ground on the Dipole Modes of Scatterers

1. Dipole modes of scatterers in free space

At low frequencies, i.e., when a target is electrically small, it is possible to represent a scatterer by short electric and magnetic dipoles. A maximum of three mutually orthogonal electric and three mutually orthogonal magnetic dipoles is required for any object, and these numbers are reduced for some shapes. The magnitude of any one dipole moment is given by the product of the parallel component of the incident field, E_O (for electric) or H_O (for magnetic), and the polarizability of the target, α. At very low frequencies, the polarizabilities are constants and the resultant scattered field has an $s^2(s = \beta + j\omega)$ Rayleigh dependence. However, if the polarizabilities are made frequency dependent the resultant scattered field is valid into the first resonance region as well as the Rayleigh region. For example, the conducting sphere of radius, a, has an electric polarizability, α_e, and a magnetic polarizability, α_m, given by

$$(46) \qquad \alpha_e(s) = \frac{K_e}{s^2 + s\,\frac{c}{a} + \left(\frac{c}{a}\right)^2}$$

$$\alpha_m(s) = \frac{K_m}{s + \frac{c}{a}} \qquad ,$$

where K_e and K_m are constants, and c is the speed of light.

Consequently, the response of the sphere has s-plane poles at $(-1/2 \pm j\sqrt{3}/2)c/a$ due to the electric dipole and at $-c/a$ due to the magnetic dipole. Other poles, farther from the origin, result from the higher order modes but the inverse transform of the three dipole mode poles, i.e., the frequency dependent scattering predicted, has been shown[11] to yield a fairly accurate ramp response waveform for the conducting sphere.

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2. Change in polarizability due to ground when dipole modes do not interact

If the scatterer is located above or within a half-space then an interaction between the target and the interface occurs which alters the target response. The simplest case to consider is that of a thin prolate spheroid which supports only an electric dipole mode along the major axis. The poles of the thin prolate spheroid as a function of eccentricity have been calculated by Page and Adams,[12] and the electric polarizability has the following form

$$(47) \qquad \alpha_e(s) = \frac{K_s}{(s+\gamma)^2 + \beta^2} \qquad ,$$

where the complex poles are located at $s = -\gamma \pm j\beta$ (γ and β real). If an electric field, $E_o(s)$, is applied parallel to the axis of the spheroid then the dipole moment, $p_e(s)$, consists of a free-space term plus an infinite sume of interaction terms

$$(48) \qquad p_e(s) = E_o(s)\, \alpha_e(s) \sum_{n=0}^{\infty} [\alpha_e(s) s\, G(s)]^n \quad ,$$

where $G(s)$ is the field scattered back from the interface to a dipole source, which in this case is the excited spheroid, for a unit current moment. The free-space term is represented by $n = 0$, and remaining terms are n-th order interactions. Ther term, s, in Eq. (48) is required because $G(s)$ is the field scattered back for a unit current moment whereas $s\, G(s)$ is the field scattered back for a unit dipole moment. Equation (48) can be summed in closed form as

$$(49) \qquad p_e(s) = E_o(s)\, \frac{\alpha_e(s)}{1 - \alpha_e(s) s\, G(s)} \qquad .$$

Consequently, the target in the presence of the interface can be considered as having a new polarizability, $\alpha_e'(s)$, given by

$$(50) \qquad \alpha_e'(s) = \frac{\alpha_e(s)}{1 - \alpha_e(s) s\, G(s)} \qquad .$$

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Thus the new polarizability depends on the target location and orientation and the half-space composition, and these effects are included in $G(s)$. As the height of the target above the interface approaches infinity, $G(s)$ goes to zero and $\alpha_e'(s)$ goes to $\alpha_e(s)$.

3. Interaction of coincident dipole modes in the presence of the ground

The new polarizability given in Eq. (50) is valid for any dipole mode which does not interact with any other dipole mode. For example, a vertical electric dipole produces only a vertical electric field and no magnetic field on the vertical axis and, as a result, will not interact with any other dipole mode. By the same reasoning, a vertical magnetic dipole mode will not interact with other dipole modes. However, horizontal electric and magnetic dipole modes which are perpendicular to each other will interact because each mode produces fields which excite the other. The perpendicular electric dipole moments, p_e, and magnetic dipole moments, p_m, can be written in the following matrix form

$$(51) \quad \begin{bmatrix} p_e \\ p_m \end{bmatrix} = \sum_{n=0}^{\infty} \begin{bmatrix} \alpha_e s \ G_{Ee} & \alpha_m s \ G_{Em} \\ \alpha_e s \ G_{He} & \alpha_m s \ G_{Hm} \end{bmatrix}^n \cdot \begin{bmatrix} \alpha_e & 0 \\ 0 & \alpha_m \end{bmatrix} \begin{bmatrix} E_o \\ H_o \end{bmatrix} \quad ,$$

where G_{Ee} is the return electric field produced by a unit electric current moment, G_{Em} is the return electric field produced by a unit magnetic current moment, G_{He} is the return magnetic field produced by a unit electric current moment, and G_{Hm} is the return magnetic field produced by a unit magnetic current moment. The matrix summation in Eq. (51) can be written in closed form as

$$\begin{bmatrix} p_e \\ p_m \end{bmatrix} = \begin{bmatrix} 1 - \alpha_e s \ G_{Ee} & - \alpha_m s \ G_{Em} \\ - \alpha_e s \ G_{He} & 1 - \alpha_m s \ G_{Hm} \end{bmatrix}^{-1} \begin{bmatrix} \alpha_e & 0 \\ 0 & \alpha_m \end{bmatrix} \begin{bmatrix} E_o \\ H_o \end{bmatrix} \quad .$$

It can be seen from Eq. (52) that each dipole mode is influenced by both the incident electric and magnetic fields in contrast with the uncoupled cases covered by Eqs. (49) and (50).

B. Use of New Dipole Modes in Target Detection

1. Use of difference equation

If a target's scattering response can be written as a rational function in s, then the inverse transform, R(t), must satisfy the following difference equation as shown by Corrington[13]

$$(53) \qquad R(t) = \sum_{k=1}^{n} (-1)^{k+1} F_{n,k} R(t - k\Delta t) \quad ,$$

where the order, n, of the difference equation is equal to the number of poles in the Laplace transform of R(t) and the difference coefficient, $F_{n,k}$, are simple functions of the poles and the time increment, Δt. The difference equation holds for all values of $t > n\Delta t$, and the time derivatives of R(t) also satisfy the same difference equation. Consequently, the ramp, step, and impulse response waveforms of a target each satisfy the same difference equation.

The exact solution for the frequency response of the conducting sphere yields an infinite number of poles, but it has been shown that the third order difference equation resulting from the three dipole-mode poles given earlier quite accurately predicts the free space ramp response of the sphere. If the dipole-mode poles of a general scatterer in free space are known, then it is possible to use Eqs. (50) and (52) to find the new poles of the scatterer in the presence of ground.

The case of a thin prolate spheroid over a perfectly conducting ground is the simplest to consider, as noted earlier, because only one electric dipole mode is involved. The polarizability in the direction of the major axis is given in Eq. (47), and the new polarizability can be found from Eq. (50). For vertical orientation of the spheroid at a height, h, above the ground, as shown in Fig. 11, G(s) can be shown to have the following form using image theory

$$(54) \qquad G(s) = \frac{s + \frac{c}{2h}}{8\pi\epsilon_0 c h^2 s} \; e^{-s\frac{2h}{c}}$$

$$G(s) \simeq \frac{1}{8\pi\epsilon_0 c^2 hs} \left[2\left(\frac{c}{2h}\right)^2 - s^2 \right] \quad ,$$
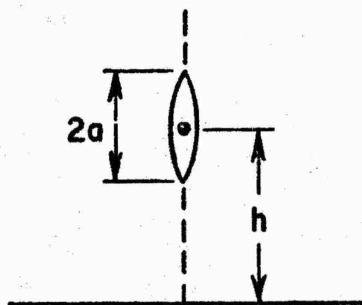
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Fig. 11. Vertical prolate spheroid above
conducting ground.

where a power series expansion of $e^{-s\,2h/c}$ has been used to obtain the
approximate form of G(s). Using Eqs. (47), (50), and (54), the new
polarizability, $\alpha_e'(s)$, is written as

$$(55) \qquad \alpha_e'(s) = \frac{K}{\left(1 + \dfrac{K}{8\pi\epsilon_o\,ch^2}\right)s^2 + 2\gamma s + \left[\gamma^2 + \beta^2 - \dfrac{2K(c/2h)^2}{8\pi\epsilon_o\,ch^2}\right]}$$

where the new poles of the spheroid-ground configuration are the zeros
of the denominator in Eq. (55). For a spheroid of semimajor axis, a,
and semiminor axis, 0.013a, the free space poles are located at
$s = (-0.1495 \pm j\,1.536)\,c/a$. This same spheroid located vertically on
the ground (h = a), has poles at $s = (-0.136 \pm j\,1.45)c/a$. The change in
the poles due to the presence of the ground is not as great as with wider
objects because the very narrow spheroid is not a strong scatterer and
the interaction with the ground is not strong.

In order to find the new poles of a conducting sphere over a ground,
it is necessary to consider both coupled and uncoupled modes. The
vertical electric and magnetic dipole modes are uncoupled, and the new
poles can be found using the method of Eq. (50). However, the orthogonal
horizontal electric and magnetic dipole modes are coupled and the new
poles of the configuration must be found from Eq. (52). Since the poles
result from the inverse matrix, they are obtained from the zeros of the
determinant:

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$$
(56) \quad
\begin{vmatrix}
1 - \alpha_e s\ G_{Ee} & - \alpha_m s\ G_{Em} \\
- \alpha_e s\ G_{He} & 1 - \alpha_m s\ G_{Hm}
\end{vmatrix}
$$

The solutions of Eq. (56) for a sphere on a conducting ground have been obtained by again taking the low frequency approximation for the G's and solving the resultant third degree equation in s. The resultant new poles for the coupled horizontal modes for a sphere are $s = -0.898\ c/a$, and $(-0.801 \pm j\, 0.905)c/a$. The new poles for the vertical electric mode of the sphere on a conductor are $s = (-0.334 \pm j\, 0.626)c/a$, and the new pole for the vertical magnetic mode is $s = -0.875\ c/a$. Consequently, the sphere-ground configuration contains a total of six poles compared to the three free-space poles given earlier, some of which may not be excited for particular excitation fields. For example, vertical polarization of the incident field will not excite the vertical magnetic mode, and horizontal polarization will not excite the vertical electric mode. To check this case, experimental backscatter measurements at four odd harmonic frequencies[14] were used to synthesize the ramp response of a sphere lying on a conducting ground. The waveform did not fit the free space difference equation well, but it did fit the free space difference equation resulting from the new poles quite well, particularly for vertical polarization.[11]

If the difference equation for a particular object is known, it can provide the following detection scheme. The received waveform is sampled in time and the difference equation of the desired object is used to predict a new waveform. If this waveform agrees closely with the received waveform, then the presence of the desired object is indicated

Note particularly that an analytical solution for the poles of the target on the ground is not a prerequisite for utilization of this predictor type processing. The method requires only that the difference equation for the ramp response waveform of a target on the ground be known. The constants $F_{n,\,k}$ in the difference equation may be determined from an analysis of experimental multiple frequency scattering data on a particular target. A rational function approximation to the complex scattering amplitude as a function of frequency is formed, wherein the order n of the difference equation is equal to the number of poles in the rational approximate and the coefficients $F_{n,\,k}$ are simple functions of the pole locations and the time increments, Δt. For spherical scatterers and a few other shapes, the pole locations can be determined analytically, and a rational approximant of any order obtained.

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The basic predictor or difference equation technique was tested by trying to detect a hemispherical boss on a conducting ground in the presence of artificial trees. Backscatter measurements at four odd harmonics[14] were used to synthesize a ramp response waveform, and the difference equation for the hemispherical boss, which is the same as the difference equation for the free space sphere, was used to predict a new waveform. The correlation of the predicted and received waveforms for vertical polarization was higher for the trees plus the boss than for the trees alone even for a fairly low signal to clutter ratio. For example, using a time increment of 0.8 c/a, the addition of the boss to the trees produced an increase in correlation from 0.382 to 0.734 when the signal to clutter power ratio was 1.72. The increase in correlation was greater for higher signal to clutter ratios and smaller for lower signal to clutter ratios. Consequently, a proper threshold setting for the correlation would permit detection of the boss in the trees. One advantage of this method is that even though the backscattered field is a function of aspect angle, the difference equation is not. Consequently, an integration on the aspect angle could be accomplished without changing the difference equation receiver.

To test the discrimination properties of the difference equation, the theoretical free-space ramp responses of spheres either larger or smaller then the desired size sphere were fed into the receiver. Using a Δt of 1.6 c/a, the correlation dropped from 1.0 to 0.5 when the sphere radius was increased 55% or decreased 20%. The correlation continues to drop as the sphere size is changed more, but the drop is more rapid with decreasing size. This is expected because it has been generally observed that rapidly varying waveforms are easier to discriminate against than slowly varying waveforms.

2. Backscattered field

The presence of the ground alters the low frequency scattering properties of an object by modifying the polarizability of the dipole modes and by changing the incident field. The first effect alone determines the shift in the poles and is sufficient for difference equation calculations which involve only the poles and not the excitation. However, if the backscattered field as a function of frequency and aspect are required as in a matched-filter receiver, then both effects are important.

To illustrate a method of combining the two effects, consider the case of a thin prolate spheroid vertically oriented at a height h above ground (Fig. 11). For a vertically polarized plane wave, the vertical component of the incident field at the center of the spheroid is given by

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$$(57) \qquad E_z(s) = E_0(s) \left[1 + R_V e^{-s \frac{2h}{c} \cos \theta} \right] \sin \theta \qquad ,$$

where θ is the aspect angle measured from the vertical and R_V is the reflection coefficient of the ground for vertical polarization. Using Eqs. (50) and (57), the dipole moment of the spheroid is

$$(58) \qquad p(s) = \frac{\alpha(s)}{1 - \alpha(s)s \ G(s)} \ E_0(s) \left[1 + R_V e^{-s \frac{2h}{c} \cos \theta} \right] \sin \theta \ .$$

The far zone backscattered field in terms of the dipole moment is

$$(59) \qquad E^S(s) = \frac{s^2 p(s)}{4 \pi \epsilon_0 c^2} \ \frac{e^{-s \, r/c}}{r} \left[1 + R_V e^{-s \frac{2h}{c} \cos \theta} \right] \sin \theta \quad .$$

Using Eqs. (58) and (59) the complex phasor response, $G^S(s)$, of the spheroid over the ground is

$$(60) \qquad G^S(s) = \frac{s^2 \sin^2 \theta}{4 \pi \epsilon_0 c^2} \ \frac{\alpha(s)}{1 - \alpha(s)s \ G(s)} \left[1 + R_V e^{-s \frac{2h}{c} \cos \theta} \right]^2 \ .$$

The two effects of the ground on $G^S(s)$ can be seen directly from Eq. (60).

The change in polarizability is represented by the factor, $[1 - \alpha(s)s \ G(s)]^{-1}$, and the altered excitation is represented by the factor, $[1 + R_V e^{-s \, 2h/c \, \cos \theta}]^2$. The problem is more complicated for objects which may require additional dipole modes, but the procedure can be extended using the modified polarizabilities in Eqs. (50) and (52).

VII. CONCLUSIONS

Exact, closed form solutions for the transient fields of vertical and horizontal electric dipoles in the presence of a half-space have been obtained for the special case of a nondispersive half-space and particular source-observer orientations. The same technique used to obtain these results can be used to obtain exact closed form solutions for vertical and horizontal magnetic dipoles under the same conditions, but these solutions have not been carried out in this report. For a nondispersive half-space

but an arbitrary observer location, the formulation leads to a finite integral which must be evaluated numerically.

If the half-space is dispersive or stratified, exact closed form solutions are not possible for the transient fields and approximate techniques must be utilized. For a dispersive half-space and observation points above or on the half-space, the form of the transient fields is such that reasonable estimates are possible from a combination of short and long time approximations. Some of the necessary approximations have been obtained using both quasi-static and asymptotic approximations and a Fourier synthesis technique.

A general model for the backscattering including interactions from electrically small (Rayleigh and low resonance range) conducting targets located on the ground has been formulated. The model illustrates that the low frequency effects of the ground on the scattering properties of the target are a modification of the polarizability of the dipole modes of the target and an alteration of the incident field. These effects have been illustrated for the case of a thin, prolate spheroid target.

A predictor type difference equation has been suggested for target detection and discrimination. The method requires that the difference equation for the ramp response of a target, either in free space or on the ground, be known and the equation can be determined from multiple frequency scattering data. For a few target shapes, the difference equation can be determined analytically. Application of the difference equation for detection has been illustrated using simulated measured data of conducting hemispherical boss targets on the ground under trees. Discrimination application has been illustrated for size separation of free space spherical scatterers.

Research on the problems discussed in this report is continuing on Contract F19628-67-C-0239. A detailed report including extended results for both the transient fields of dipoles in the presence of a half-space and applications of the difference equation to detection and discrimination is anticipated in the near future.

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13. ABSTRACT

The transient fields produced by vertical and horizontal electric dipoles with impulsive current excitation in the presence of a half-space are considered. For special source and observer configurations, exact, closed form expressions for the fields produced by impulsively excited dipoles above a nondispersive half-space are given. For the dispersive case, estimates of the transient fields from asymptotic approximations are given. A knowledge of the interactions of dipole antennas and a half-space is useful for studies of the background return from the half-space and for studies of the scattering by small targets on or near the half-space.

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